NEW TYPES OF REGULAR CONVERGENCE

By Paul A. White

G. T. Whyburn [4] defined a new type of convergence which he called "regular" by approximating the condition of local connectedness in the members of the converging sequence. Thus the convergence $(M_n) \to M$ was called r-regular if corresponding to any $\epsilon > 0$ there exists a $\delta > 0$ and N such that for $n > N$ any Vietoris cycle in M_n with diameter $< \delta$ is ~ 0 in a subset of M_n with diameter $< \delta$. In this paper new types of regular convergence are studied in which the condition of local connectedness is replaced by other local properties. In particular a generalization of semi-local connectedness as introduced by G. T. Whyburn [6] is used to define r-dimensional coregular convergence, and the property complete r-avoidability introduced by R. L. Wilder [8] is used to define r-dimensional completely avoidable regular convergence (r-c.a. regular). A study is first made of the properties of the limit set under these types of convergence and it is shown that if the convergence is i-regular for $i \leq r - 1$, and r-coregular, then the limit set contains no r-cut points (see Theorem 2.1). Also if the convergence is i-regular for $i \leq r$, and r-c.a. regular, then the limit set is completely r-avoidable at each point (see Theorem 2.2). Finally, if the convergence is i-regular for $i \leq n$, i-c.a. regular for $i \leq n - 2$, and $(n - 1)$-coregular, then the limit of a sequence of n-dimensional closed Cantorian Manifolds is an n-dimensional generalized manifold in the sense of R. L. Wilder [7]. In the last section a study of the 0-dimensional convergences is made and it is shown that 0-c.a. regular convergence implies both 0-regular and 0-coregular convergence for continua. It is shown that a sequence of 2-dimensional compact manifolds converges 0-c.a. regularly to another 2-dimensional compact manifold (or a point). If each member of the sequence has n disjoint simple closed curves as boundary, then the limit set has $m \leq n$ simple closed curves as its boundary. If each member of the sequence is a sphere with n handles, then the limit set is a sphere with $m \leq n$ handles.

It will be assumed throughout that all sets used lie in a compactum. All of the ordinary complexes and cycles will have mod 2 coefficients and all Vietoris cycles (V-cycles) will have these as coördinates. $S(A, \epsilon)$ shall denote the set of all points x such that the distance from x to $A(\rho(x, A))$ is $< \epsilon$; $F(A, \epsilon)$, all x such that $\rho(x, A) = \epsilon$.

1. **General properties of limiting sets.** The following two lemmas and theorem are generalizations of Lemmas (1.1), (1.2), and Theorem 1.4 of the paper [4].

Lemma 1.1. *Let the sequence of closed sets* $(A_n) \to A$. *If a set of numbers* $e > d_1 > 3d_2 > c \geq 0$ *is given such that for any* $\epsilon > 0$ *there exist numbers* $\delta > 0$

Received December 22, 1944.