A GENERALIZATION OF THE SEXTACTIC POINT OF A PLANE CURVE

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1. In a series of recent papers [1], [2], [3], and [4] we have considered a singular point of a plane curve such that the tangent at this point of the curve has with the curve a contact of order greater than one. On the other hand E. P. Lane [5] has treated the sextactic point of a plane curve, a singularity for which the osculating conic, taken in place of the tangent, has a contact of order five with the curve. After defining an invariant point analogous to Halphen point he has determined the canonical expansion of the curve by means of the neighborhood of order eight. In projective differential geometry of curves we are often bound to consider the canonical expansion determined by the neighborhood of order seven instead of eight. A new method is proposed here for this purpose.

When the osculating conic becomes a k ($k \geq 6$)-point conic at O we call O a *k-ic* point. Although Lane's method of determining Halphen point fails for $k > 6$, our method remains applicable for *7-ic* and *8-ic* points.

2. Let a point in a plane be given by non-homogeneous coördinates x, y; then the homogeneous coördinates x_1, x_2, x_0 may be defined by

$$x = \frac{x_1}{x_0}, \qquad y = \frac{x_2}{x_0}.$$

We select first the triangle of reference such that the vertex $O(0, 0, 1)$ is the sextactic point of the given curve C and the side $y = 0$ the tangent of the curve at O. If $(0, 1, 0)$ be taken on the osculating conic, then the curve C can be represented in the neighborhood of O by a power series expansion of the form

$$y = ax^2 + ex^6 + fx^7 + gx^8 + hx^9 + (10), \tag{1}$$

where $ae \neq 0$.

Through a given point $A(1, 0, \alpha)$ on $y = 0$ we can draw besides $y = 0$, another tangent

$$4a\alpha x_1 - 4ax_0 - \alpha^2 x_2 = 0 \tag{2}$$

to the osculating conic

$$x_0 x_2 - ax_1^2 = 0. \tag{3}$$

Received September 6, 1944. The stay of the author in the National University of Chekiang was made possible by the Board of Trustees of the Indemnity Funds remitted by the British Government.