# NEW GEOMETRICAL CHARACTERIZATIONS OF SOME SPECIAL CONJUGATE NETS

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1. **Introduction.** Let $N_x$ be a conjugate net with parameters $u$, $v$ on an analytic proper surface $S$ in ordinary space, and let $x_{-1}$, $x_1$ be respectively the ray-points, or Laplace transformed points, of a general point $P_x$ of the surface $S$ with respect to the $v$-curve and the $u$-curve of the conjugate net $N_x$. As $u$, $v$ vary the loci of the points $x_{-1}$, $x_1$ are two surfaces $S_{-1}$, $S_1$ on which the parametric curves form two conjugate nets $N_{-1}$, $N_1$, which are called as usual the minus-first and first Laplace transformed nets of $N_x$, respectively. The purpose of the present paper is to give new geometrical characterizations of the conjugate net $N_x$ in some special cases by introducing two quadrics, called *the associated quadrics at the point $P_x$ of the conjugate net $N_x$*, one of which has contact of the second order with the surface $S_{-1}$ at the point $x_{-1}$ and contact of the first order with the surface $S_1$ at the point $x_1$, and the other has similar properties with the roles of $u$, $v$ interchanged.

2. **The associated quadrics.** Let $N_x$ be a conjugate net with parameters $u$, $v$ on an analytic proper surface $S$ in ordinary space. For the sake of convenience we take the conjugate net $N_x$ on the surface $S$ as parametric, so that the projective homogeneous coördinates $x^{(1)}, \cdots, x^{(4)}$ of a point $P_x$ on the surface $S$ can be given as analytic functions of the two independent variables $u$, $v$ by equations of the form

$$x = x(u, v). \tag{1}$$

The four coördinates $x$ and the four coördinates $y$ of the point $P_y$ which is the harmonic conjugate of the point $P_x$ with respect to the foci of the axis of the point $P_x$ satisfy a system of partial differential equations of the form [2; 138]

$$\begin{aligned} x_{uu} &= px + \alpha x_u + Ly, \\ x_{uv} &= cx + ax_u + bx_v, \\ x_{vv} &= qx + \delta x_v + Ny \qquad (LN \neq 0), \end{aligned} \tag{2}$$

subscripts indicating partial differentiation and the coefficients being functions of $u$, $v$ which satisfy certain integrability conditions. It is easy to verify that

$$y_u = fx - nx_u + sx_v + Ay, \qquad y_v = gx + tx_u + nx_v + By, \tag{3}$$

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