SOLUTIONS OF SYSTEMS OF DIFFERENTIAL EQUATIONS IN THE VICINITY OF BRANCH POINTS OF THE SOLUTIONS, III

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Introduction. The system of differential equations to be considered in this paper has the form

$$(1) \qquad x_1^{\alpha_{1i}} x_2^{\alpha_{2i}} \cdots x_n^{\alpha_{ni}} \frac{dx_i}{dt} = \frac{a_i}{m+1} + f_i(t) + \sum_{\nu=1}^{\infty} f_{i\mu}^{(\nu)} x_1^{\mu_1} x_2^{\mu_2} \cdots x_n^{\mu_n} \qquad (i = 1, \cdots, n),$$

where m is a positive integer, $\mu_1, \mu_2, \cdots, \mu_n$ are non-negative integers, μ represents the sequence $\mu_1, \mu_2, \cdots \mu_n$, $\nu = \mu_1 + \mu_2 + \cdots + \mu_n$, $a_i \neq 0$ and

$$(2) \qquad \sum_{j=1}^{n} \alpha_{ji} = m \qquad (i = 1, \cdots, n).$$

The functions f_i, $f_{i\mu}^{(\nu)}$ are assumed to have the following properties:

I. The functions f_i, $f_{i\mu}^{(\nu)}$ are integrable on the straight line in the t-plane from t_0 to the point t. The length of this straight line will be represented by u.

II. The functions f_i, $f_{i\mu}^{(\nu)}$ satisfy the inequalities

$$(3) \qquad \begin{cases} |f_i| \leq \dfrac{Mnu^{1/(m+1)}}{m+1} \\ \left|\dfrac{f_{i\mu}^{(\nu)}}{c_i}\right| \leq M \\ |b_i| \leq A \end{cases},$$

where the b_i and c_i are defined by equations (9) and (11) respectively and A and M are positive constants and $A > 1$.

1. **Formal solution of the system of differential equations** (1). The transformation

$$(4) \qquad x_i = \sum_{h=1}^{\infty} K^h y_{ih}$$

reduces the system (1) to the form

$$(5) \qquad \begin{aligned} K^{m+1} y_{11}^{\alpha_{1i}} y_{21}^{\alpha_{2i}} \cdots y_{n1}^{\alpha_{ni}} \frac{dy_{i1}}{dt} + \sum_{h=2}^{\infty} K^{m+h} \left[y_{11}^{\alpha_{1i}} y_{21}^{\alpha_{2i}} \cdots y_{n1}^{\alpha_{ni}} \frac{dy_{ih}}{dt} + \varphi_{ih}(y_{lk}, y'_{lk}) \right] \\ = \frac{a_i}{m+1} + f_i(t) + \sum_{h=2}^{\infty} K^{h-1} g_{ih}(t, y_{lk}) \\ \left(i, l = 1, \cdots, n; k = 1, \cdots, h-1; y'_{lk} = \frac{dy_{lk}}{dt}\right), \end{aligned}$$

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