NÖRLUND SUMMABILITY OF MULTIPLE FOURIER SERIES

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1. **Introduction.** Let $x = (x_1, \cdots, x_k)$ be a point in the k-dimensional Euclidean space and let $f(x) = f(x_1, \cdots, x_k)$ be a Lebesgue integrable function having period 2π in each variable. Let its Fourier series be

$$f(x) \sim \sum a_{n_1 \cdots n_k} e^{i(n_1 x_1 + \cdots + n_k x_k)}, \tag{1.1}$$

where

$$a_{n_1 \cdots n_k} = \frac{1}{(2\pi)^k} \int_{-\pi}^{\pi} \cdots \int_{-\pi}^{\pi} f(x) e^{-i(n_1 x_1 + \cdots + n_k x_k)}\, dx. \tag{1.2}$$

In this paper we shall consider both the rectangular partial sums

$$s_{N_1 \cdots N_k}(x; f) = \sum_{n_1 = -N_1}^{N_1} \cdots \sum_{n_k = -N_k}^{N_k} a_{n_1 \cdots n_k} e^{i(n_1 x_1 + \cdots + n_k x_k)} \tag{1.3}$$

and also the "triangular" partial sums

$$T_N^0(x; f) = \sum_{\nu \le N} a_{n_1 \cdots n_k} e^{i(n_1 x_1 + \cdots + n_k x_k)} \qquad (\nu = |\, n_1 \,| + \cdots + |\, n_k \,|). \tag{1.4}$$

The reason for calling these partial sums "triangular" is evident if we consider the double Fourier series and observe the shape of the array of terms for n_1, $n_2 \ge 0$.

We shall consider chiefly Nörlund [8] methods of summability. To define the Nörlund transform of any simple sequence $\{y_n\}$, let $\{p_n\}$ be any sequence of constants. Let

$$P_n = \sum_{k=0}^{n} p_k \ne 0.$$

The Nörlund mean of $\{y_n\}$ is

$$t_n = \frac{1}{P_n} \sum_{k=0}^{n} p_{n-k} y_k . \tag{1.5}$$

If t_n tends to a finite limit as $n \to \infty$, the sequence $\{y_n\}$ is said to be summable N_p to this limit. We shall consider only regular Nörlund methods of summability. The conditions of regularity for N_p are [4; 758]

$$\sum_{k=0}^{n} |\, p_k \,| = O(|\, P_n \,|), \qquad p_n/P_n \to 0 \text{ as } n \to \infty. \tag{1.6}$$

Cesàro (C, α), $\alpha \ge 0$, is clearly a regular Nörlund method, for in this case $P_0 = 1$, $P_n = A_n^\alpha = (\alpha + 1) \cdots (\alpha + n)/n!$ for $n \ge 1$.

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