SOME REMARKS ON THE PROBLEM OF GEÖCZE

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1. The Lebesgue area $A(S)$ of a continuous surface S is defined as follows:

$$A(S) = \text{gr.l.b. lim inf } E(P_n), \tag{1}$$

where P_n is a sequence of polyhedra converging to S in the Fréchet sense, $E(P_n)$ is the area of P_n in the elementary sense, and the greatest lower bound is taken with respect to all sequences of polyhedra P_n converging to S. For the precise definition of the various terms involved, see for example [3]. Note that the approximating polyhedra P_n are *not* required to be inscribed in S. Let $A^*(S)$ be the quantity obtained by requiring, in the definition of $A(S)$, that the polyhedra P_n *be inscribed* in S. Clearly,

$$A^*(S) \geq A(S). \tag{2}$$

The problem of Geöcze consists of deciding whether or not the sign of equality holds. If $A(S) = \infty$, then surely $A^*(S) = A(S)$ by (2). Thus we are justified, in discussing the problem of Geöcze, to assume

$$A(S) < \infty. \tag{3}$$

2. The problem of Geöcze in its general form is apparently beyond the reach of the methods available at present. On the other hand, the literature contains a considerable amount of valuable results concerning special types of surfaces S, especially surfaces given by an equation $z = f(x, y)$. In a general way the complexity of the methods used seems to be unreasonable in view of the restricted character of the results achieved. By contrast, a recent paper by H. D. Huskey [2], concerned with the case of surfaces of the form $z = f(x, y)$, is quite remarkable first because the results obtained are far more general than those previously obtained, and second on account of the surprising simplicity of the method. The method of Huskey is based upon a recent important paper of L. C. Young [6] on the area of surfaces. The purpose of this note is to develop further the relationships between the integral considered by Young and the area of inscribed polyhedra, along the lines suggested by the work of Huskey. To avoid repetitions the topics discussed and the results obtained will be summarized at the end of this note, in §17.

3. Let there be given a continuous surface

$$S : z = f(x, y), \tag{4}$$

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