THE CESÀRO SUMMABILITY OF THE CONJUGATE SERIES OF A FOURIER SERIES

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1. All the classical criteria [3], [5], [12], [15] for the convergence of a Fourier series are included in a theorem of Lebesgue [6] which was generalized by Pollard [10]. In 1930, Gergen [4] obtained a criterion more general than Pollard's.

As for the convergence of the conjugate series of a Fourier series, theorems corresponding to those of [3], [5], [12], [15] are established by [11], [13], [14], [9]. A criterion corresponding to Lebesgue's is given by [7]. Recently, Chen [2] obtained a convergence criterion for the conjugate series of a Fourier series which corresponds to that of Gergen for Fourier series. The object of this note is to extend Chen's theorem of convergence into a criterion for the Cesàro summability with positive order.

Let

$$\sum_{n=1}^{\infty} (b_n \cos nx - a_n \sin nx) \tag{1.1}$$

be the conjugate series of the Fourier series corresponding to the function $f(x)$ which is integrable in the sense of Lebesgue over the interval $(-\pi, \pi)$ and is defined outside this interval by periodicity. If we assume the existence of the integral

$$g(x) = \frac{1}{2\pi} \int_0^{\pi} \psi(t) \cot \tfrac{1}{2}t \, dt \tag{1.2}$$

with

$$\psi(t) = f(x + t) - f(x - t)$$

in the sense of Cauchy, Chen's criterion states that (1.1) converges to $g(x)$ provided that

$$\lim_{k\to\infty} \overline{\lim_{\eta\to 0}} \int_{k\eta}^{\pi} \frac{|\Delta_\eta \psi(t)|}{t} \, dt = 0,$$

where $\Delta_\eta \psi(t) = \psi(t + \eta) - \psi(t)$.

Writing

$$\Delta_\eta^{(m)} \psi(t) = \sum_{\nu=0}^{m} (-1)^{m+\nu} \binom{m}{\nu} \psi(t + \nu\eta),$$

we will prove the following theorem:

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