INTERIOR MAPPINGS INTO THE CIRCLE

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1. **Introduction.** In fulfillment of an earlier prediction [4] it will be shown in this paper that any continuous transformation of a cyclic locally connected continuum into the circle is homotopic to an interior transformation. Thus in this situation each homotopy mapping class contains an interior mapping which could serve as its representative. Also in §3 a somewhat more inclusive theorem applying to non-cyclic locally connected continua is obtained.

It will be understood that a continuum is a compact connected metric space. A continuous transformation $f(x)$ of a continuum A onto a set B is (a) *interior* (or *open*) provided the image of every open set in A is open in B, (b) *monotone* if the inverse of each point of B is connected, (c) a *retraction* if $B \subset A$ and $f(x) = x$ for $x \varepsilon B$, and (d) *quasi-monotone* if, for each continuum K in B with a non-empty interior, $f^{-1}(K)$ has just a finite number of components and each of these maps onto K under f. If B is a circle S and A is locally connected it was shown in [4] that f is quasi-monotone if and only if, for each $y \varepsilon S$, each component of $A - f^{-1}(y)$ maps onto $S - y$ under f.

For convenience we will let S denote the unit circle $|z| = 1$ in a complex plane and shall employ ordinary multiplication of complex numbers on S. Also if U is any open set, $F(U)$ will denote its boundary, i.e., $F(U) = \overline{U} - U$.

2. Theorem. *Any continuous mapping of a cyclic locally connected continuum M into the circle is homotopic to an interior mapping.*

Proof. Let $f(M) = S$ be continuous. By a previous theorem of the author [4], f is homotopic to a quasi-monotone transformation. Hence there is no loss in generality in supposing f is quasi-monotone.

Now since (cf. [5]) f is interior at all points of $f^{-1}(S - C)$, where C is some countable subset of S, there exists a $z \varepsilon S$ such that f is interior at all points of $f^{-1}(z) + f^{-1}(-z)$. Let

$$a = z, \qquad b = -z, \qquad A = f^{-1}(a), \qquad B = f^{-1}(b).$$

Let axb and ayb be the two semicircles into which a and b divide S and let

$$X = f^{-1}(axb), \qquad Y = f^{-1}(ayb).$$

Let N be the hyperspace of the decomposition of M into the sets A and B and the individual points of $M - (A + B)$ and let $g(M) = N$ be the transformation thus generated. We further define

$$g(X) = X', \qquad g(Y) = Y', \qquad g(A) = a', \qquad g(B) = b'.$$

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