REPRESENTATIONS OF FINITE SIMPLE SEMIGROUPS

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By a semigroup is meant a system $\mathfrak{S}$ of elements $a, b, \cdots$ closed under an associative binary operation:

$$(ab)c = a(bc).$$

The problem of representing semigroups as correspondences of a set to itself has not been treated in the literature and the first part of this paper is devoted to a study of this problem in the finite case. Chapter I deals with semigroups of correspondences of a set N to itself and culminates in the result that N has a unique decomposition as the union of "strongly transitive" systems, so-called because of their likeness to the transitive systems of permutation group theory. It is then possible to regard a representation of a semigroup as composed of a collection of semigroups on strongly transitive systems, and, as outlined in Chapter II, the representation problem reduces to that of finding all such basic representation semigroups together with methods for compounding them.

The major part of the paper, Chapters III and IV, is devoted to the construction of all representations of a type of finite semigroup which is the sum of a set of isomorphic groups and called a Kerngruppe by Suschkewitsch [2], or, in the terminology of Rees [1], a finite simple semigroup without zero. The fundamental representation theorem, in the terminology of Chapter III, is:

Each transitive representation of a finite simple semigroup $\mathfrak{S}$ is equivalent to a representation on the classes of a subpartition of the basic partition of the set of left cosets determined by a right group $\mathfrak{R}$ in $\mathfrak{S}$. Conversely, any right group and a subpartition of the basic partition of its left cosets determines a transitive representation of $\mathfrak{S}$.

In the event $\mathfrak{S}$ is a group, $\mathfrak{R}$ is a subgroup, the partition classes become the various cosets of $\mathfrak{R}$, and the result specializes to the theorem characterizing the transitive representations of a group. In Chapter IV it is shown how all other representations of the semigroups under consideration can be constructed from transitive ones.

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