PLANE SECTIONS OF CERTAIN RULED SURFACES ASSOCIATED WITH A CURVED SURFACE

By Chuan-Chih Hsiung

1. **Introduction.** Let P_x be a general point of an analytic non-ruled surface S referred to its asymptotic net in ordinary projective space, and let C be any curve on the surface S through the point P_x. As the point P_x moves along the curve C, the tangents u, v generate two ruled surfaces R_u, R_v respectively. If a general plane π intersects the asymptotic tangents u, v at P_x of the surface S respectively in two points T, T^*, then the latter are simple points of the plane sections Γ_u, Γ_v of the two ruled surfaces R_u, R_v made by π. The purpose of the present paper is to study such plane sections in detail.

§2 contains power series expansions of the plane sections Γ_u, Γ_v which are used in later developments. In §3 a new transformation of Čech is obtained from the polarity between TT^* and a line through P_x. In the last section we continue to find the loci of certain osculating conics of the plane sections Γ_u, Γ_v at the points T, T^*, and then derive a one-parameter family of cones of the sixth class.

2. **Power series expansions.** Let the surface S under consideration be an analytic non-ruled surface in ordinary space. We employ the notation of [3; 69, 71, 79] and consider a curve C imbedded in the one-parameter family of curves defined on the surface S by the equation

$$dv - \lambda\, du = 0. \tag{1}$$

The u-tangent at a point X near the point P_x on the curve C is determined by X, X_u, whose non-homogeneous coördinates are

$$\begin{aligned} \xi_1 &= \Delta u + \tfrac{1}{2}(\theta_u + \gamma\lambda^2)\Delta u^2 + \cdots, \\ \eta_1 &= \lambda\Delta u + \tfrac{1}{2}(\beta + \theta_v\lambda^2 + \lambda')\Delta u^2 + \cdots, \\ \zeta_1 &= \lambda\Delta u^2 + \cdots, \end{aligned} \tag{2}$$

$$\begin{aligned} \xi_2 &= \frac{1}{p\Delta u}\left[1 + \left(\theta_u - \frac{G}{2p}\right)\Delta u + \cdots\right], \\ \eta_2 &= \frac{1}{p\Delta u}\left\{\beta\Delta u \right. \\ &\quad \left. + \tfrac{1}{2}\left[\beta_u + \beta\theta_u + 2(p + \beta\psi)\lambda + (\beta\gamma + \theta_{uv})\lambda^2 - \frac{\beta G}{p}\right]\Delta u^2 + \cdots\right\}, \\ \zeta_2 &= \frac{1}{p\Delta u}\left[\lambda\Delta u + \tfrac{1}{2}\left(\beta + 2\theta_u\lambda + \theta_v\lambda^2 + \lambda' - \frac{G}{p}\lambda\right)\Delta u^2 + \cdots\right], \end{aligned} \tag{3}$$

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