THE ASYMPTOTIC FORMS OF THE SOLUTIONS OF AN ORDINARY LINEAR MATRIC DIFFERENTIAL EQUATION IN THE COMPLEX DOMAIN (CONTINUED)

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Introduction. In an earlier paper [2], I discussed the existence of solutions to the matric differential equation

$$\frac{d}{dx} Y(x, \lambda) = \{\lambda(\delta_{ij}r_j(x)) + (q_{ij}(x, \lambda))\} Y(x, \lambda),$$

in which x and λ are complex variables. Under certain conditions of suitability imposed upon the coefficient functions and the domains of variation of x and λ [2; §2], it was possible to show that regions of existence can be constructed in which the differential equation possesses solutions of the form $P(x, \lambda)E(x, \lambda)$, where $E(x, \lambda) = (\delta_{ij} \exp\{\lambda \int^x r_j(x)\, dx\})$, and $P(x, \lambda)$, analytic in x, reduces uniformly in x to the identity matrix when λ becomes infinite.

Specifically, I applied the elegant and basic concepts of associated and fundamental regions originated by R. E. Langer [1; §3] to some of the cases in which the functions $r_j(x)$ have poles and the differences $r_i(x) - r_j(x)$ $(i \neq j)$ have poles or zeros on the boundary of the x region in question. Langer had considered previously the case in which the coefficient functions $r_j(x)$ and $q_{ij}(x, \lambda)$ are all analytic and bounded as to x, and the differences $r_i(x) - r_j(x)$ $(i \neq j)$ are all bounded from zero [1; 2]. In adapting the concept of associated regions to the cases in question, it seemed necessary to impose the condition upon the differences $r_i(x) - r_j(x)$ $(i \neq j)$ that those having simple poles be of the form $a^{ij}(x - x_0)^{-1}$, where a^{ij} $(\neq 0)$ is a constant [2; §4, Theorem 1]. It is the purpose of this paper to remove that restriction.

There is little to separate the present discussion from the former one; hence the notation and terminology adopted there will be retained. It will also be convenient to use here a continuation of the numbering and sectioning of the earlier paper.

8. **Associated and fundamental regions.** Suppose that x and λ range over regions of the complex plane which are suitable to equation (2.1) of [2] in the sense of the definition given in §2 of [2]. As noted in §3 of [2], there exists a set of functions $R_j(x)$ $(j = 1, \cdots, n)$ which are analytic except possibly for poles and logarithmic infinities on the boundary of the suitable x region, and which satisfy the relations [2]

$$\frac{d}{dx} R_j(x) \equiv r_j(x) \qquad (j = 1, \cdots, n). \tag{3.1}$$

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