CERTAIN QUANTITIES TRANSCENDENTAL OVER $GF(p^n, x)$, II

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1. Let $GF(q)$ denote a fixed finite field of order $q = p^n$. Let x be an indeterminate over $GF(q)$ and denote by $GF[q, x]$ the ring of polynomials in x with coefficients from the finite field. $GF(q, x)$ will be the quotient field of $GF[q, x]$. We are concerned here with the transcendence of certain quantities over $GF(q, x)$.

Place

$$[k] = x^{q^k} - x, \qquad F_k = [k][k-1]^q \cdots [1]^{q^{k-1}},$$

$$F_0 = 1, \qquad L_k = [k] \cdots [1], \qquad L_0 = 1.$$

L. Carlitz [1] has studied the function

$$\psi(t) = \sum_{i=0}^{\infty} (-1)^i \frac{t^{q^i}}{F_i}$$

and its inverse

$$\lambda(t) = \sum_{i=0}^{\infty} \frac{t^{q^i}}{L_i}.$$

(For convergence, see [1].) In particular, there is a quantity $\xi \neq 0$ (in a suitable field containing $GF(q, x)$) such that

$$\psi(E\xi) = 0$$

for all polynomials E, i.e., all elements of $GF[q, x]$. It was proved in a previous paper [3] that if $\alpha \neq 0$ is algebraic over $GF(q, x)$, then $\psi(\alpha)$ is transcendental. In particular, ξ is transcendental.

Here we shall prove the transcendence of

$$\sum_{i=0}^{\infty} \frac{1}{L_i^{\gamma}}$$

when γ is a positive rational integer. This will enable us to give a new proof of the transcendence of ξ. The theorem could be generalized slightly and similar theorems proved by the same method.

2. We will use deg as an abbreviation for degree. If $E \neq 0$ and $G \neq 0$ are two polynomials over $GF(q)$, we define

$$\deg \frac{E}{G} = \deg E - \deg G.$$

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