PROJECTIVE DIFFERENTIAL GEOMETRY OF A PAIR OF PLANE CURVES

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1. **Introduction.** There are many possible cases for the relative position of two curves in a plane. The case which will be considered in the following lines is that in which the two curves have a common tangent at two ordinary points, which we shall call *ordinary points of the second kind* of the curves. On the contrary, if two ordinary points of two plane curves be in a general position, they are called ordinary points of the first kind. Bouton [2] has obtained a projective invariant determined by the neighborhood of the second order of two plane curves at two ordinary points of the first kind.

The purpose of this paper is to study the projective differential geometry of two real plane curves in the neighborhood of two ordinary points of the second kind.

In §2, we obtain two projective invariants, which are determined by the neighborhood of the fourth order of the two curves at the two ordinary points of the second kind.

In §3 by introducing a certain covariant triangle of reference associated with the two curves at the two ordinary points of the second kind, we reduce the invariants obtained in §2 to a simpler form, and give them geometrical characterizations.

In §4, we suitably choose a covariant point for the unit point of the coördinate system, so that the canonical power series expansions of the two curves at the two ordinary points of the second kind are obtained. According as the two invariants obtained in §2 vanish or not, we have four different types. The interpretation of the absolute invariants in the expansions of every type is given in terms of certain cross ratios alone.

In the last section we show that the covariant lines under consideration may be regarded as generalizations of a covariant line which Bompiani [1] associated with the point of contact of two tangent plane curves.

2. **Two projective invariants.** Suppose that O_1 and O_2 are two ordinary points of the second kind of two plane curves C and C^* respectively, so that O_1O_2 is the common tangent. Let the homogeneous projective coördinates of a point in the plane be denoted by (x_1, x_2, x_3). If the points O_1 and O_2 are chosen respectively for the vertices (1, 0, 0) and (0, 1, 0) of the triangle of reference, the power series expansions of the two curves in the neighborhood of the points O_1 and O_2 may be written in the form

$$(1) \qquad C: \quad \frac{x_3}{x_1} = a_1\left(\frac{x_2}{x_1}\right)^2 + a_2\left(\frac{x_2}{x_1}\right)^3 + a_3\left(\frac{x_2}{x_1}\right)^4 + a_4\left(\frac{x_2}{x_1}\right)^5 + a_5\left(\frac{x_2}{x_1}\right)^6 + \cdots,$$

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