ASYMPTOTIC RULED SURFACES

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1. **Introduction.** The directrices of Wilczynski [21] are among the interesting pairs of covariant lines associated with a point M of a surface S immersed in ordinary projective space. The first directrix passes through the point M and does not lie in the tangent plane at the point M of the surface S, and the second lies in the tangent plane of S at M but does not pass through M. Suppose that the first directrix intersects the quadric of Lie at another point M_3, and the second directrix intersects the asymptotic tangents u, v at points M_1, M_2 respectively.

If we consider the ruled surface of the v-tangents constructed at the points of the asymptotic curve u through M, or as we shall say, for brevity, *the asymptotic ruled surface* Σ_u, the other asymptotic tangents of Σ_u at M and M_2 coincide with MM_1 and M_2M_3 respectively, and therefore the plane section of Σ_u made by any plane through MM_1 or M_2M_3 has at M or M_2 an inflexion. Similarly, the plane section of the asymptotic ruled surface Σ_v made by any plane through MM_2 or M_1M_3 has also an inflexion at M or M_1. In §3, we shall make use of the points of Bompiani [2] representing the neighborhood of the fourth order of these plane sections and therewith derive two sequences of covariant quadrics from the quadric of Lie of the surface S at the point M.

There are two quadrics Q_u and Q_v of Moutard of the asymptotic ruled surfaces Σ_u and Σ_v at M belonging to a non-asymptotic tangent of the surface S at the point M. In §4, we propose to study these quadrics Q_u and Q_v in detail (obtaining the equations of Q_u and Q_v by a method somewhat different from that of Cech [9; 529–533]), giving new interpretations for the scroll directrices of Sullivan [20; 202] and some canonical lines as well as an analogue of the cone of Segre and the transformation of Cech [6; 192].

If two plane curves C_1 and C_2 have a contact of the second order at a point 0, there is associated a covariant line $r_0^{(2)}$ of Bompiani [3], which the author [12] has geometrically characterized. Since the plane sections of the asymptotic ruled surfaces Σ_u and Σ_v made by any plane through a non-asymptotic tangent of the surface S have, in general, a contact of the second order at the point M, we may therefore apply the considerations just mentioned. This forms the content of §5.

By a line l_1 we mean any line through the general point M of the surface S and not lying in the tangent plane of S at M, and by a line l_2 we mean, dually, any line in the tangent plane of S at M but not passing through the point M. G. Palozzi [15] has constructed a correspondence between l_1 and l_2 by means of the plane section of the surface S made by the plane through one of the asymptotic tangents and by means of the point of Bompiani on this tangent. This suggests that we use the asymptotic ruled surfaces of S at M instead of S.

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