THE ASYMPTOTIC FORMS OF THE SOLUTIONS OF AN ORDINARY LINEAR MATRIC DIFFERENTIAL EQUATION IN THE COMPLEX DOMAIN

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1. **Introduction.** The matric differential equation

$$\frac{d}{dx} Y(x, \lambda) = \{\lambda(\delta_{ij} r_j(x)) + (q_{ij}(x, \lambda))\} Y(x, \lambda),$$

under conditions to be given below, has solutions of the form $P(x, \lambda)E(x, \lambda)$, where $E(x, \lambda) = (\delta_{ij} \exp \{\lambda \int^x r_j(x)\, dx\})$ and $P(x, \lambda)$, analytic in x, reduces uniformly in x to the identity matrix when λ becomes infinite.

The present discussion rests directly upon, and extends, theory recently published by R. E. Langer[1] who showed that if the coefficient functions $r_j(x)$ are all analytic and bounded in a region of the complex plane and their differences $r_i(x) - r_j(x)$ $(i \neq j)$ are all bounded from zero and if, moreover, the functions $q_{ij}(x, \lambda)$ are analytic in x, bounded in x and λ and, for $|\lambda|$ large, admit either actual or asymptotic expansions in λ^{-1} with coefficients analytic and bounded in x, then a solution of the form $P(x, \lambda)E(x, \lambda)$ exists for the above differential equation in the neighborhood of any specified point of the given x region. This paper, for the most part, deals with equations in which the coefficient functions $r_j(x)$ may have poles and the differences $r_i(x) - r_j(x)$ $(i \neq j)$ may have zeros on the boundary of the x region in question. Regions of existence which abut such a pole or zero are established for a solution of the stated form $P(x, \lambda)E(x, \lambda)$.

In addition, it is shown that the restriction to finite x regions, assumed in Langer's discussion, may be removed.

2. **The matric equation.** Throughout the considerations to follow, the differential equation[2]

$$\frac{d}{dx} Y(x, \lambda) = \{\lambda R(x) + Q(x, \lambda)\} Y(x, \lambda), \tag{2.1}$$

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[1] R. E. Langer, *The boundary problem of an ordinary linear differential system in the complex domain*, Trans. of the Am. Math. Soc., vol. 46(1939), pp. 151–162.

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[2] In the notation adopted here, italic capitals without subscripts denote square matrices of order n. The operations of differentiation and integration are applied in accordance with the relations

$$\frac{d}{dx}(y_{ij}(x)) = \left(\frac{d}{dx} y_{ij}(x)\right), \qquad \int^x (y_{ij}(x))\, dx = \left(\int^x y_{ij}(x)\, dx\right)$$

in which the right members serve to define the left. Also, a matrix is said to be analytic if each element is analytic.