FUNDAMENTAL THEOREMS OF A NEW MATHEMATICAL THEORY OF PLASTICITY

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1. **Introduction.** The mathematical theory of plasticity was inaugurated in 1871 by B. de Saint Venant. The progress it has made since then is much smaller than that of the mathematical theory of elasticity in the period of almost equal length between Cauchy's fundamental researches and the first edition of Love's treatise. The main reason for this comparatively slow progress seems to be the tremendous mathematical difficulty arising from the assumption that the material will not behave in a plastic manner unless a certain invariant of the stress tensor has reached a given critical value. Alongside plastic regions we will, therefore, have low-stressed regions in which the material is not yet plastic and behaves elastically. Two different sets of equations are valid in the plastic and the elastic regions and the problem becomes all the more involved by the fact that the boundary between these regions is not known beforehand but has to be determined so as to secure continuity of stresses.

In order to avoid this great difficulty the author has proposed stress-strain relations which give a *gradual* transition from the elastic to the plastic state (Proc. 5th Intern. Congr. Appl. Mech., Cambridge, Mass., 1938, p. 234). In a recent paper the simplest of these stress-strain relations has been applied to various problems of plane strain (Revue Fac. Sci., Univ. Istanbul, ser. A, vol. 5, p. 215). The present paper contains two variational principles which, in this new mathematical theory of plasticity, play the same rôle as Castigliano's principle and the principle of least work do in elasticity.

2. **Stress-strain relation.** As long as an elastic region subsists, strains in the plastic regions will be of the same order of magnitude as those in the elastic region. As in elasticity we assume these strains to be infinitesimal.

Let σ_{ik} and ϵ_{ik} be the components of the tensors of stress and strain with respect to a set of rectangular axes. In order to simplify our equations we shall assume the material to be incompressible. Adopting the summation convention for repeated indices, generally used in tensor calculus, we write the condition of incompressibility in the form

$$\epsilon_{pp} = 0. \tag{1}$$

Introducing

$$\delta_{ik} = \begin{cases} 0 & \text{if } i \neq k, \\ 1 & \text{if } i = k, \end{cases}$$

and defining the mean normal stress as

$$\sigma = \tfrac{1}{3}\sigma_{pp}\,, \tag{2}$$

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