ABSOLUTE NÖRLUND SUMMABILITY

By Leonard McFadden

1. **Introduction.** The method of summability considered here was first introduced by Woronoi [11],[1] but is more closely identified with the name of Nörlund [8]. Accordingly, we will use the term "Nörlund transformations" to indicate the transformations in question.

Let $\{p_n\}$ be a sequence of constants, real or complex valued, and let $\{P_n\}$ denote the sequence of partial sums. We will call $\{t_n\}$ the Nörlund transform of an arbitrary sequence $\{U_n\}$, where

$$(1.01) \qquad t_n = \sum_{v=0}^{n} \frac{p_{n-v} U_v}{P_n},$$

it being assumed of course that $P_n \neq 0$. The sequence $\{U_n\}$ is said to be summable by the Nörlund mean N_p defined by $\{p_n\}$, or summable N_p, if $\lim_{n \to \infty} t_n$ exists.

The conditions for regularity of such a transformation are

$$(1.02) \qquad \lim_{n \to \infty} \frac{p_n}{P_n} = 0,$$

$$(1.03) \qquad \sum_{k=0}^{n} | p_k | \leqq C | P_n |,$$

where C is a finite positive constant. It is easily seen that (1.02) is equivalent to

$$(1.04) \qquad \lim_{n \to \infty} \frac{P_{n-1}}{P_n} = 1.$$

We might also note that, if p_n is real and non-negative, condition (1.03) is satisfied automatically and, if in addition p_n is non-increasing, condition (1.02) is also satisfied.

The sequence $\{U_n\}$ will be said to be absolutely summable by the Nörlund mean defined by the sequence $\{p_n\}$, or summable $| N_p |$, provided that

$$(1.05) \qquad \sum_{n=1}^{\infty} | t_n - t_{n-1} | \leqq C < \infty.$$

If we let N_p and $| N_p |$ denote, respectively, the class of sequences summable N_p and $| N_p |$, we have the following

(1.06) Theorem. $| N_p | \subset N_p$; $N_p \not\subset | N_p |$.

Received November 3, 1941.

[1] Numbers in brackets refer to the bibliography at the end of the paper.