INEQUALITIES FOR TRIGONOMETRIC INTEGRALS

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Introduction. We are concerned with the following problem: *If*

$$f(x) = \int_{-R}^{R} e^{ixt}\, ds(t), \tag{0.1}$$

$$g(x) = \int_{-R}^{R} \mu(t) e^{ixt}\, ds(t), \tag{0.2}$$

where $s(t)$ is a complex valued function of bounded variation on $[-R, R]$ and $\mu(t)$ is continuous, and if

$$|f(x)| \leqq M \qquad (-\infty < x < \infty), \tag{0.3}$$

then find a bound for $|g(x)|$.

The present paper is divided into two parts. In Part I, the general problem is considered for various classes of functions $\mu(t)$, sufficiently restricted so that non-trivial bounds for the corresponding functions can be found. Part II is devoted to the case in which $\mu(t) = (it)^\alpha$, $0 < \alpha < 1$, so that $g(x)$ is the fractional derivative of $f(x)$ of order α.

We shall consistently use the symbols appearing in (0.1), (0.2), and (0.3) with the meanings which they have in these formulas.

Part I

The following lemma gives the fundamental method of approach to the problem. We adopt the notation

$$p(n) = n\pi/R \qquad (n = 0, \pm 1, \pm 2, \cdots).$$

Lemma 1. *If*

$$\mu(t) e^{iat} = \sum_{-\infty}^{\infty} c_n e^{ip(n)t}, \tag{1.1}$$

where a is a real number, then

$$|g(x)| \leqq M \sum_{-\infty}^{\infty} |c_n| \qquad (-\infty < x < \infty). \tag{1.2}$$

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