AN ANALOGUE OF THE BERNOULLI POLYNOMIALS

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1. We define a set of polynomials $\beta_m(u)$ with coefficients in $GF(p^n, x)$ by means of[1]

$$\frac{\psi(tu)}{u\psi(t)} = \sum_{m=0}^{\infty} \frac{\beta_m(u)}{g_m} t^m. \tag{1.1}$$

The function $\psi(t)$ may be defined by[2]

$$\psi(t) = \sum_{i=0}^{\infty} (-1)^i \frac{t^{p^{ni}}}{F_i}, \tag{1.2}$$

where

$$F_k = (x^{p^{nk}} - x)(x^{p^{nk}} - x^{p^n}) \cdots (x^{p^{nk}} - x^{p^{n(k-1)}}), \qquad F_0 = 1;$$

and the denominator in the right member of (1.1) is given by

$$g_m = g(m) = F_0^{a_0} F_1^{a_1} \cdots F_s^{a_s}, \tag{1.3}$$

where

$$m = a_0 + a_1 p^n + \cdots + a_s p^{ns} \qquad (0 \leqq a_i < p^n).$$

Thus it is clear that the coefficients in $\beta_m(u)$ are rational functions over the $GF(p^n)$, indeed over the $GF(p)$.

If we put

$$\frac{t}{\psi(t)} = \sum_{m=0}^{\infty} \frac{B_m}{g_m} t^m, \tag{1.4}$$

then by (1.1) and (1.2)

$$\beta_m(u) = \sum_{p^{ni} \leqq m+1} (-1)^i \frac{g(m)}{F_i g(m - p^{ni} + 1)} B_{m-p^{ni}+1} u^{p^{ni}-1}. \tag{1.5}$$

On the other hand, if we use the expansion

$$\psi(tu) = \sum_{i=0}^{\infty} (-1)^i \frac{\psi_i(u)}{F_i} \psi^{p^{ni}}(t), \tag{1.6}$$

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[1] The right member of (1.1) contains only terms for which $p^n - 1 \mid m$; $\beta_m(u)$ is defined for such m only.

[2] For the properties of $\psi(t)$ and $\psi_k(t)$ used here see L. Carlitz, *On certain functions connected with polynomials in a Galois field*, this Journal, vol. 1(1935), pp. 137–168.