# RIEMANNIAN MANIFOLDS WITH POSITIVE MEAN CURVATURE

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1. Intuitively it is natural to expect that a manifold whose curvature is everywhere positive and bounded away from zero will, if indefinitely extended, be closed. Illustrating this geometric idea, in 1931 Hopf and Rinow[1] proved that a complete surface whose curvature is everywhere greater than or equal to a positive constant $e^2$ is closed (compact) and has a diameter not exceeding $\pi/e$, and its universal covering manifold is also closed with diameter not exceeding $\pi/e$. In 1935[2] the present author generalized this result to complete $n$-dimensional Riemannian manifolds. There the curvature $K$ is a function of a point and a plane of directions; and if $K \geqq e^2$ for all points and all planes of directions, the space is closed and has diameter not exceeding $\pi/e$, the same results holding for the universal covering manifold.

The question now arises as to what conclusions can be drawn if the curvature is allowed to vary more freely on a complete manifold, but the mean curvature,[3] a function of a point and a direction, is kept $\geqq e^2$. The result of the present paper is that such a manifold $M$ is closed and has[4] diameter not exceeding $a$, where

$$a = \frac{\pi\sqrt{n - 1}}{e};$$

also, its universal covering manifold is closed and has diameter not exceeding $a$. This implies that the fundamental group of $M$ is finite. An important application of this result is to spaces of constant positive mean curvature, which are solutions of the field equations in the general theory of relativity.[5]

2. Let $M$ be an $n$-dimensional Riemannian manifold[6] of class $C^{(3)}$, whose mean curvature everywhere $\geqq e^2$. We prove first the following lemma.

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[1] H. Hopf and W. Rinow, *Über den Begriff der vollständigen differential-geometrischen Fläche*, Comm. Math. Helvetici, vol. 3(1931), p. 224.

[2] S. B. Myers, *Riemannian manifolds in the large*, this Journal, vol. 1(1935), pp. 42–43.

[3] The mean curvature at a point $P$ with respect to a vector $(v)$ is the sum of the $n - 1$ 2-dimensional curvatures obtained by pairing $(v)$ with each of a set of $n - 1$ mutually orthogonal vectors all orthogonal to $(v)$, and is independent of the choice of such a set. See Eisenhart, *Riemannian Geometry*, p. 113.

[4] If mean curvature were defined as an average instead of a sum, then this upper bound for diameter would be simply $\pi/e$ instead of $a$.

[5] The results of this paper are, however, proved for positive definite metrics, and hence are not immediately applicable to the physical theory.

[6] See, for example, Myers, op. cit., p. 40.