THE JUMP OF ALMOST PERIODIC FUNCTIONS AND OF FOURIER INTEGRALS

BY OTTO SZÁSZ

1. We have proved recently [2][1] the following:

THEOREM A. *Suppose that $f(t)$ is integrable L in $(-\pi, \pi)$, and has period 2π; if for a fixed x there exists a $D(x)$ such that*

$$(1) \qquad \int_0^h |f(x+t) - f(x-t) - D(x)|\, dt = O(h) \quad \text{as } h \downarrow 0$$

and

$$(2) \qquad \int_0^h \{f(x+t) - f(x-t) - D(x)\}\, dt = o(h) \quad \text{as } h \downarrow 0,$$

then

$$\frac{1}{2n}\{\sum_{\nu=1}^{n} \nu B_\nu(x) + \sum_{\nu=n+1}^{2n}(2n-\nu)B_\nu(x)\} \to \frac{D(x)\log 2}{\pi} \quad \text{as } n \uparrow \infty;$$

here

$$\sum_{\nu=1}^{n} B_\nu(t) = \sum_{1}^{n}(b_\nu \cos \nu t - a_\nu \sin \nu t) \qquad (n = 1, 2, 3, \cdots)$$

are the partial sums of the conjugate Fourier series corresponding to $f(t)$.

$D(x)$ is the generalized jump of $f(t)$ at a point x; if in particular $f(x+t) - f(x-t) \to D(x)$ as $t \downarrow 0$, then (1) and (2) obviously hold.

We shall give here analogous results for generalized Fourier series and for Fourier integrals.

2. We consider real-valued almost-periodic functions in the sense of Besicovitch ([1], Chapter II). If $f(t)$ is a B.a.p. function, then

$$M\{f(t)e^{-i\lambda t}\} = \lim_{h\uparrow\infty} \frac{1}{2h}\int_{-h}^{h} f(t)e^{-i\lambda t}\, dt = c(\lambda)$$

exists for all real values of λ, and it may differ from zero for at most an enumerable set of values λ ([1], Chapter II, §8). Of these let the positive λ be arranged in some order:

$$\lambda_1, \lambda_2, \lambda_3, \cdots, \lambda_n > 0,$$

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[1] Numbers in brackets refer to the bibliography at the end of this paper.