ORDERLY DIFFERENTIAL SYSTEMS

BY JOSEPH MILLER THOMAS

1. **Introduction.** Riquier has given two general existence theorems for systems of partial differential equations in a field of holomorphic functions. The first of these concerns orthonomic systems [8, 254].[1] Because it is locally applicable without any restriction being placed on the holomorphic functions defining the equations and the initial determination, it can conveniently be called an unrestricted theorem. The second is a restricted theorem generalizing the first. It is applicable both to orthonomic systems and to some non-orthonomic systems whose defining functions and initial determinations are subjected to certain inequalities [8, 384, 387].

The present paper defines an orderly system (§27) as a system decomposable into a finite number of orthonomic systems and proves for such a system an unrestricted existence theorem (Theorem 29.1). The class of orderly systems thus includes the orthonomic as a proper subclass. It also contains a subclass for which Riquier's non-orthonomic theorem does not give even a restricted result.

Without seriously complicating the analysis the method used here can be employed to prove a restricted theorem for a class of systems including as a proper subclass all systems covered by Riquier's two theorems. In order not to complicate the ideas now to be presented, the discussion of this generalization will be postponed.

The proof of the theorem for orderly systems is believed to have advantages over those previously given for orthonomic systems [2], [8], [9, 135–156], [13]. For this reason it seems desirable to make the present treatment self-contained. The chief feature is the absence of the auxiliary integers called cotes by Riquier. Their elimination makes available a simple direct means for testing whether a given system is in orderly or in orthonomic form. The corresponding test for orthonomic systems defined in the old manner is known, but its application involves a somewhat extended knowledge of the theory of linear inequalities [10].

2. **Notation.** The unknowns and independent variables will be denoted respectively by u_α ($\alpha = 1, 2, \cdots, r$) and x_i ($i = 1, 2, \cdots, n$), where r, n are arbitrary fixed positive integers. For the derivative

$$\frac{\partial^{i_1+i_2+\cdots+i_n} u_\alpha}{\partial x_1^{i_1}\, \partial x_2^{i_2} \cdots \partial x_n^{i_n}} \tag{2.1}$$

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[1] References to the bibliography at the end of the paper are given in brackets. The first number designates the entry and subsequent numbers, not otherwise described, the pages.