FUNCTIONS GENERATED BY DEVELOPING POWER SERIES IN CONTINUED FRACTIONS

By A. Markoff

This paper, although published long ago, remains unknown, evidently because of language difficulties. It contains important results concerning zeros of orthogonal polynomials, results susceptible of further developments. In view of the ever-growing interest in the field of orthogonal polynomials, it seems desirable to make this work of Markoff accessible to a larger group of readers by means of an English translation. The translator has adhered faithfully to the original, except for employing Perron's notation for continued fractions, the abbreviation $[\alpha_{i+j}]_0^{2n-2}$ for the determinant

$$\begin{vmatrix} \alpha_0 & \alpha_1 & \cdots & \alpha_{n-1} \\ \alpha_1 & \alpha_2 & \cdots & \alpha_n \\ \cdots & \cdots & \cdots & \cdots \\ \alpha_{n-1} & \alpha_n & \cdots & \alpha_{2n-2} \end{vmatrix},$$

and ${}_{i,j}[\alpha_{i+j}]_0^{2n-2}$ for the same determinant with the column α_i, α_{i+1}, $\cdots$ and the row α_j; α_{j+1}, $\cdots$ crossed out. In addition, he has numbered formulas and theorems in order to facilitate references.

Let

$$\frac{S_0}{x} + \frac{S_1}{x^2} + \frac{S_2}{x^3} + \cdots \tag{1}$$

be a power series arranged according to integral negative powers of the variable x.

By successive divisions it is possible to transform it, as is known, into the continued fraction

$$\frac{1|}{|q_1} - \frac{1|}{|q_2} - \cdots, \tag{2}$$

where q_1, q_2, $\cdots$ are polynomials in x.

These polynomials evidently depend also on the parameters S_0, S_1, S_2, $\cdots$.

We assign to the parameters

$$S_0, S_1, \cdots, S_{2m-2}, S_{2m-1} \tag{3}$$

real values for which none of the determinants

$$\Delta_1 = S_0, \qquad \Delta_2 = \begin{vmatrix} S_0 & S_1 \\ S_1 & S_2 \end{vmatrix}, \qquad \cdots, \qquad \Delta_m = [S_{p+q}]_0^{2m-2}$$

vanishes.

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