A NOTE ON STRONGLY IRREDUCIBLE MAPS OF AN INTERVAL

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A continuous transformation of the compact metric space A onto $B, f(A) = B$, is called *strongly irreducible*[1] provided no proper closed subset of A maps onto all of B. Clearly, any continuous transformation (map) of a compact metric space A into B is strongly irreducible on *some* closed subset A^1 of A. On the other hand, there exists no strongly irreducible map of an *interval* onto a continuum of the form of the letter T. The purpose of this note is to give a proof that a certain extensive class of Peano spaces can be obtained by strongly irreducible maps of the interval. Under the condition given below we not only obtain the desired mapping of the interval I onto the Peano space M but also permit the specification of a certain dense subset of M *in advance* on which the inverse of the mapping function will be single valued.

The letter L will denote throughout the set of local separating points of the Peano continuum M.

THEOREM. *Let M be a Peano continuum such that $M \subset \overline{M - L}$. If P is any countable dense set of non-local separating points of M, there exists a continuous function f such that: $f(I) = M$, where I is the unit interval; if $y \in P$, $f^{-1}(y)$ is a single point; and $\overline{f^{-1}(P)} \supset I$. Thus f is a strongly irreducible map of I onto M.*[2]

Denote by M^I the class of maps of I into subsets of M and suppose it is metrized in the usual way by $\sigma(f,g) = \underset{x \in I}{\text{l.u.b.}}\, \rho[f(x), g(x)]$ $(f, g \in M^I)$. The subset J consisting of maps of I onto all of M constitutes a closed subset of the complete space M^I. By the Hahn-Mazurkiewicz Theorem, $J \neq 0$. Henceforth J will be regarded as our function space. Evidently, J is complete. It will be shown that the class J^* of maps of I onto M which satisfies the conclusions of our theorem is a dense G_δ set in J.

LEMMA 1. *Let M be a Peano continuum such that $M \subset \overline{M - L}$. Let P be any countable dense set of non-local separating points of M. The set $H \subset J$ such that, for $f \in H$, $f^{-1}(y)$ is single valued for each $y \in P$ is a dense G_δ.*

Proof. Set $P = p_1 + p_2 + p_3 + \cdots$, $P_n = p_1 + p_2 + \cdots + p_n$. Denote by F_n^k the subset of J such that for some point of P_n the map has two inverses x^1 and x^2 with $|x^1 - x^2| \geqq 1/k$. The set F_n^k is closed. Put $F_n = \sum_{k=1}^{\infty} F_n^k$. Evidently, F_n is merely the subset of J such that f^{-1} is not single valued on

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[1] See G. T. Whyburn, *On irreducibility of transformations*, American Journal of Mathematics, vol. 61(1939), p. 820, and references given therein.

[2] Ibid., Theorem 2.