SYLOW THEOREMS FOR INFINITE GROUPS

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There exists a class of results in the theory of finite groups whose proofs are obtained by the method of counting; i.e., one shows that, unless the theorem in question holds true, there is an impossible abundance—or scarcity for that matter—of subsystems with some property. The results obtained this way may be called "Sylow theorems" after their most important representative. Typical examples are the following facts: The existence of central-elements, different from 1, in finite p-groups $\neq 1$; the conjugacy of any two greatest p-subgroups of a finite group; P. Hall's discussion of Sylow-systems of finite soluble groups and so on.

Though trying to break away from the classical limitation to the investigation of finite groups, one may still employ the methods used in the proofs of theorems of the Sylow-type to obtain results concerning groups which are restricted in no way as to size. Certain conditions concerning the subsystems investigated turn out to be needed for the applicability of these methods. They are, however, in general not necessary for the validity of these extensions of theorems of the Sylow-type. But in imposing the condition that finite subsets are contained in finite normal subgroups, a class of groups has been characterized for which a fairly complete theory may be evolved.[1]

Chapter I. Sylow subgroups

1. The only theorem concerning finite groups which we are going to use in this chapter is the *theorem of Cauchy* stating that *a finite group contains an element of order the prime number p if, and only if, the order of the group is divisible by p.* All the other theorems on finite groups, excepting elementary ones, which will be used will be proved as special cases of theorems on groups which may be finite or infinite.

We enumerate some of the notations we are going to use. If S is a subset and x an element of the group G, then $S^x = x^{-1}Sx$. Two subsets U and V of G are termed W-conjugate, where W is also a subset of G, if there exists an element w in W so that $U = V^w$. This relation is symmetric, reflexive and transitive, whenever W is a subgroup of G. U^W signifies the set of all the U^w for w in W.

If S is a subgroup of the group G, then $(S < G)$ is the normalizer of S in G

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[1] In this connection the following papers ought to be mentioned: A. P. Dietzmann, *Über p-Gruppen*, Comptes Rendus de l'Académie des Sciences de l'URSS, vol. 15(1937), pp. 71–76; A. P. Dietzmann, A. Kurosch, A. I. Uzkow, *Sylow-sche Untergruppen von unendlichen Gruppen*, Recueil Mathématique, vol. 3(1938), pp. 179–184.