FUNCTIONS OF SEVERAL VARIABLES AND ABSOLUTE CONTINUITY, II

BY C. B. MORREY, JR.

In Part I of this paper, by J. W. Calkin [2],[1] functions of class $\mathfrak{P}$, $\mathfrak{P}'$, $\mathfrak{P}''$, $\mathfrak{P}_\alpha$, $\mathfrak{P}'_\alpha$, and $\mathfrak{P}''_\alpha$ were defined and various theorems were proved concerning these functions. The object of this part is to carry forward the study of these functions to the point where they may be used in the study of standard problems of analysis, particularly those problems in which the authors are interested. Part I of this paper comprises §§1–5 and the present part contains §§6–9 inclusive. We shall not refer to Part I by name but will merely refer to certain theorems, lemmas, and definitions therein, such as Theorem 3.5, etc.; in other words we shall regard the two parts as a single unit.

Certain notations were introduced in §1 which simplify the discussion considerably. We shall continue the use of these notations and shall present them before proceeding with the discussion. The n-tuples $(x_1, \cdots, x_n)$, $(\xi_1, \cdots, \xi_n)$, etc., are denoted by the single letters x, ξ, etc. The closed cell $a_i \leqq x_i \leqq b_i$ $(i = 1, \cdots, n)$ is denoted by $[a, b]$ and the open cell by (a, b). The functional notation $f(x_1, \cdots, x_n)$ is abbreviated to $f(x)$ and the Lebesgue integrals of $f(x)$ over (a, b) and a set E are denoted respectively by

$$\int_a^b f(x)\,dx, \qquad \int_E f(x)\,dx.$$

It is frequently desirable to consider the behavior of $f(x)$ with reference to a particular variable x_k or the $n - 1$ variables $(x_1, \cdots, x_{k-1}, x_{k+1}, \cdots, x_n)$. In such a case, we write x_k' for $(x_1, \cdots, x_{k-1}, x_{k+1}, \cdots, x_n)$, (x_k', x_k) for x, and $f(x_k', x_k)$ for $f(x)$. Thus, if the coördinates x_k' are fixed with $x_k' = a_k'$, $f(a_k', x_k)$ denotes the function $f(a_1, \cdots, a_{k-1}, x_k, a_{k+1}, \cdots, a_n)$ of the single variable x_k. Similarly, if x_k is fixed with $x_k = a_k$, $f(x_k', a_k)$ denotes the function $f(x_1, \cdots, x_{k-1}, a_k, x_{k+1}, \cdots, x_n)$ of the $n - 1$ variables $(x_1, \cdots, x_{k-1}, x_{k+1}, \cdots, x_n) = x_k'$. We denote the projection of the closed cell $[a, b]$ on $x_k = 0$ by $[a_k', b_k']$ and that of (a, b) by (a_k', b_k'). The integral

$$\int_{a_k'}^{b_k'} f(x_k', a_k)\,dx_k'$$

denotes the $(n - 1)$-dimensional integral of $f(x_k', a_k)$ over the cell (a_k', b_k'), i.e., over the $(n - 1)$-cell $a_1 < x_1 < b_1, \cdots, a_{k-1} < x_{k-1} < b_{k-1}, a_{k+1} < x_{k+1} <$

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[1] Numbers in brackets refer to the bibliography.