REGULAR TRANSFORMATIONS

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1. Let M be a continuum and $T(M) = M'$ be an $(r - 1)$-regular transformation.[1] It is shown in this paper that if a', b' are any two points of M', then the s-dimensional Betti groups of $T^{-1}(a')$ and $T^{-1}(b')$ relative to M are isomorphic for $s = 0, 1, \cdots, r$. Furthermore, in case T is a monotone 0-regular transformation, it is shown that the 1-dimensional Betti group of M is the direct sum of two groups, one of which is isomorphic with the 1-dimensional Betti group of M', while the other is isomorphic with the 1-dimensional Betti group of $T^{-1}(a')$ relative to M for any point a' of M'. Thus $p^1(M) = p^1(M') + p^1(T^{-1}(a'), M')$, where $p^1(N)$ is the first Betti number of N and, for any point a' of M', $p^1(T^{-1}(a'), M)$ is the number of linearly independent cycles in $T^{-1}(a')$ relative to homologies in M.

The cycles and bounding relations used here are with respect to an arbitrary modulus $m \geqq 0$. The combinatorial notions used will be found in works of Alexandroff[2] and Vietoris.[3] After the convention of Alexandroff, $z^r \simeq 0 \pmod m$ indicates that the cycle z^r bounds an $(r + 1)$-dimensional complex relative to m, while $z^r \sim 0 \pmod m$ indicates that there exists a number α such that αz^r bounds. In case $m \geqq 2$, the two relations are the same.

2. Let the sequence of closed sets $\{A_n\}$, which are contained in a compact metric space M, converge to the limiting set A. The sequence is said to converge *r-regularly* (mod m) provided that for each $\epsilon > 0$ there exist positive numbers δ and N such that if $n > N$, any r-dimensional potentially bounding true cycle[4] (mod m) in A_n of diameter $< \delta$ is $\simeq 0 \pmod m$ in a subset of A_n of diameter $< \epsilon$. The convergence here defined differs from that given by G. T. Whyburn[5]

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[1] The transformation used here is a generalization of the 0-regular transformation defined by A. D. Wallace, Bulletin of the American Mathematical Society, abstract 44-3-161.

[2] *Dimensionstheorie*, Mathematische Annalen, vol. 106(1932), pp. 161–238. For elementary combinatorial notions see also P. Alexandroff and H. Hopf, *Topologie* I, Berlin, 1935.

[3] *Über den höheren Zusammenhang kompakter Räume* $\cdots$, Mathematische Annalen, vol. 97(1927), pp. 545–572.

[4] A true cycle $Z^r = (z_1^r, z_2^r, z_3^r, \cdots)$ is said to be potentially bounding provided all except a finite number of the z^r are potentially bounding (mod m). If r is > 0, any z^r is potentially bounding, while if $r = 0$, z^0 is potentially bounding if and only if the coefficient sum is $\equiv 0 \pmod m$.

[5] *On sequences and limiting sets*, Fundamenta Mathematicae, vol. 25(1935), pp. 408–426. It may be pointed out here that if the convergence is 0-regular for any m, it is 0-regular for every m, therefore is called simply 0-regular.