THE EXISTENCE OF CERTAIN TRANSFORMATIONS

By G. T. Whyburn

The general existence problem for a specified type of transformation of one given compact set onto another is of long standing and has received numerous contributions over a considerable span of years. For example, a classical result of Hahn and Mazurkiewicz yields the conclusion that any compact locally connected continuum can be mapped continuously onto any other one but cannot be so mapped onto a non-locally-connected continuum.

In this paper the question of the mappability of a compact locally connected continuum M onto an interval by particular sorts of continuous transformations will be considered. This is, of course, the same thing as considering the definability of particular kinds of continuous, real-valued functions on M. In this connection the reader is referred to closely related papers by Čech,[1] Mazurkiewicz,[2] Aitchison,[3] C. Pauc,[4] Kuratowski,[5] and the author.[6]

We consider monotone, non-alternating, interior and light transformations. If A and B are compact continua, a continuous transformation $T(A) = B$ is (1) *monotone*[7] provided the inverse set $T^{-1}(b)$ of each point b in B is connected, (2) *non-alternating*[8] if for any two points x and y of B, $T^{-1}(x)$ does not separate any two points of $T^{-1}(y)$ in A, (3) *interior*[9] provided the image of every set open in A is open in B, and (4) *light* provided that for each point b in B, $T^{-1}(b)$ is totally disconnected (or of dimension 0).

The principal results will be found in §§2 and 4. In §2 it is shown that a compact locally connected continuum M can be mapped onto an interval by a non-alternating interior transformation f if and only if the cyclic elements of M are arranged into a cyclic chain. In §4 it is shown that in case M is 1-dimensional, f can in addition be chosen as a light transformation.

1. **Lemmas on joining and subdivision.** If a and b are points of a locally connected continuum M and K is the set of all points separating a and b in M,

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[1] Fundamenta Mathematicae, vol. 18(1932), p. 85.

[2] Ibid., p. 88.

[3] Comptes Rendus de la Société des Sciences et des Lettres de Varsovie, vol. 27(1934).

[4] Comptes Rendus, Paris, vol. 202(1939), p. 489.

[5] Fundamenta Mathematicae, vol. 30(1938), p. 17.

[6] American Journal of Mathematics, vol. 55(1933), p. 131.

[7] See R. L. Moore, Transactions of the American Mathematical Society, vol. 27(1925), p. 416; C. B. Morrey, American Journal of Mathematics, vol. 57(1935), p. 17; also reference in footnote 8.

[8] See G. T. Whyburn, American Journal of Mathematics, vol. 56(1934), pp. 394–402.

[9] See Stoïlow, Annales Scientifiques de l'Ecole Normale Supérieure, vol. 63(1928), pp. 347–382.