CONFIGURATIONS DEFINED BY THETA FUNCTIONS

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The aspects of theta function theory which I wish to present today have come to my own attention in connection with an effort to develop an invariant theory for the Cremona group in a projective space which would in some sense parallel the invariant theory of the projective group in that space. One type of such a theory arises in the plane in connection with the idea of a set of n points P_n^2 as the carrier of the totality Σ_n^2 of all the complete linear systems of curves which can be defined by their multiplicities at the points of the set. This duality between P_n^2 and Σ_n^2 is obviously invariant under projective transformation, i.e., if P_n^2 is carried into $P_n'^2$ by a projectivity π, then Σ_n^2 defined by P_n^2 is carried by π into $\Sigma_n'^2$ defined by $P_n'^2$. But it is also true that if P_n^2 and $P_n'^2$ are such that $n - \rho$ pairs p_i, p_i' drawn from the two sets are corresponding pairs of a Cremona transformation τ for which the remaining ρ points in each set are the direct and inverse F-points of τ, then also Σ_n^2 defined by P_n^2 is carried by τ into $\Sigma_n'^2$ defined by $P_n'^2$. Under these circumstances we say that the set P_n^2 is *congruent* to the set $P_n'^2$ under the Cremona transformation τ and regard this notion of congruence of sets of points P_n^2 as the extension to the Cremona group of the notion of projectivity of sets of points. In this notion of congruence, as in projectivity, the *order* of the points in the related sets P_n^2, $P_n'^2$ is obviously material.

When we extend this notion to sets of points P_n^k in spaces of higher dimension k, it is necessary to confine the Cremona transformations to elements of the "regular" Cremona group, i.e., to transformations which can be defined by "isolated" F-points—transformations which are termed "punctual" by Miss Hudson.

In developing this notion of congruence, we find it convenient to eliminate projectivity by using the obvious canonical form in which the first $k + 2$ points of P_n^k are taken to be the reference points, and the unit point and the factors of proportionality in the coördinates of the remaining $n - k - 2$ points are so adjusted that the last coördinate of each is the same, say u. Then the set P_n^k is uniquely determined by the coördinates of a point P in a space $S_{k(n-k-2)}$,

$$P: \quad x_{ij}:x_{i'j'}: \cdots :u \qquad (i, i', \cdots = k + 3, \cdots, n; j, j', \cdots = 1, \cdots, k).$$

The ratios $x_{ij}:u$ are then absolute projective invariants or double ratios determined by P_n^k. Naturally this mapping of sets P_n^k in S_k on points P of

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