

IDENTITIES ANALOGOUS TO RAMANUJAN'S IDENTITIES INVOLVING THE PARTITION FUNCTION

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1. **Introduction.**¹ A new proof of the two identities of Ramanujan

$$(1.11) \quad \sum_{n=0}^{\infty} p(5n+4)x^n = 5 \frac{\varphi(x^5)^5}{\varphi(x)^6},$$

$$(1.12) \quad \sum_{n=0}^{\infty} p(7n+5)x^n = 7 \frac{\varphi(x^7)^3}{\varphi(x)^4} + 49x \frac{\varphi(x^7)^7}{\varphi(x)^8},$$

where $p(n)$ is the number of unrestricted partitions of n and $\varphi(x) = \prod_{n=1}^{\infty} (1-x^n)$, has recently been given in a paper by Rademacher and the author.² The method used is not limited to verifying known identities, but can be extended to obtain new ones. In this paper the following new identities will be proved:

$$(1.13) \quad \sum_{n=0}^{\infty} p(25n+24)x^n = 63 \cdot 5^2 \frac{\varphi(x^5)^6}{\varphi(x)^7} + 52 \cdot 5^5 x \frac{\varphi(x^5)^{12}}{\varphi(x)^{13}} + 63 \cdot 5^7 x^2 \frac{\varphi(x^5)^{18}}{\varphi(x)^{19}} \\ + 6 \cdot 5^{10} x^3 \frac{\varphi(x^5)^{24}}{\varphi(x)^{25}} + 5^{12} x^4 \frac{\varphi(x^5)^{30}}{\varphi(x)^{31}},$$

$$(1.14) \quad \sum_{n=0}^{\infty} p(49n+47)x^n = 2546 \cdot 7^2 \frac{\varphi(x^7)^4}{\varphi(x)^5} + 48934 \cdot 7^4 x \frac{\varphi(x^7)^8}{\varphi(x)^9} + 1418989 \cdot 7^6 x^2 \frac{\varphi(x^7)^{12}}{\varphi(x)^{13}} \\ + 2488800 \cdot 7^7 x^3 \frac{\varphi(x^7)^{16}}{\varphi(x)^{17}} + 2394438 \cdot 7^9 x^4 \frac{\varphi(x^7)^{20}}{\varphi(x)^{21}} \\ + 1437047 \cdot 7^{11} x^5 \frac{\varphi(x^7)^{24}}{\varphi(x)^{25}} + 4043313 \cdot 7^{12} x^6 \frac{\varphi(x^7)^{28}}{\varphi(x)^{29}} \\ + 161744 \cdot 7^{15} x^7 \frac{\varphi(x^7)^{32}}{\varphi(x)^{33}} + 32136 \cdot 7^{17} x^8 \frac{\varphi(x^7)^{36}}{\varphi(x)^{37}} \\ + 31734 \cdot 7^{18} x^9 \frac{\varphi(x^7)^{40}}{\varphi(x)^{41}} + 3120 \cdot 7^{20} x^{10} \frac{\varphi(x^7)^{44}}{\varphi(x)^{45}} \\ + 204 \cdot 7^{22} x^{11} \frac{\varphi(x^7)^{48}}{\varphi(x)^{49}} + 8 \cdot 7^{24} x^{12} \frac{\varphi(x^7)^{52}}{\varphi(x)^{53}} + 7^{25} x^{13} \frac{\varphi(x^7)^{56}}{\varphi(x)^{57}},$$

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¹ Many of the results of this paper have been obtained by G. N. Watson in a paper (*Journal für Mathematik*, vol. 179(1938), pp. 97-128) which appeared since this was written. The methods of the two papers are quite different.

² A new proof of two of Ramanujan's identities, to be published in the *Annals of Mathematics*.