SIMULTANEOUS INVARIANTS OF A COMPLEX AND SUBCOMPLEX

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Part I. Introduction

1. Let P^n (of dimension n) and Q be finite Euclidean polyhedra with $Q \subset P = P^n$. (We use the definitions and notation of P. Alexandroff and H. Hopf, *Topologie*, I.) Moreover, let P and Q admit of a *permissible simplicial division* K_{-1}, i.e., a simplicial division of P such that some subcomplex of K_{-1}, say L_{-1}, is a simplicial division of Q (it is not assumed that the simplexes of K_{-1} are Euclidean simplexes). Let K_1 and L_1 denote the second proper barycentric subdivisions of K_{-1} and L_{-1}, respectively (the word proper indicates that centers of gravity are used). We define the *neighborhood* N_1 of L_1 to be the simplicial complex consisting of the simplexes of K_1 that have at least one vertex in L_1 (together with the sides of all such simplexes).

This neighborhood N_1 depends upon the permissible simplicial division employed. However, we prove in this paper that certain properties of N_1 are independent of the permissible simplicial division.

In the first place, we prove that the structures of the following groups are invariant when the permissible division is changed: (a) the Betti groups (with respect to a pair of coefficient domains as in Alexandroff-Hopf, p. 205) of the simplicial complex B_1 consisting of the simplexes of N_1 that have no vertex in L_1, (b) the Betti groups of the simplicial complex R_1 consisting of the simplexes of K_1 that have no vertex in L_1, (c) groups consisting of homology classes of B_1 that bound in R_1 (cf. $H^r(B_1, R_1)$ of Alexandroff-Hopf, p. 345), and (d) groups consisting of homology classes of B_1 that bound both in R_1 and N_1 (cf. $N^r(N_1, R_1)$ of Alexandroff-Hopf, p. 292, 8). For the definitions of groups (c) and (d) see §8. A relation among the groups is given in §9.

These results have already found applications in the study of generalized and singular manifolds.

The essential feature of the proof is the construction of a set of rays (called deformed rays) which are conveniently related to the subdivision of N_1 and along which cycles can be homotopically deformed.

Part II. The deformed rays

2. **The straight rays.** In §2 we study K_{-1}, a fixed permissible simplicial division of P and Q. We assume in this section that K_{-1} is a Euclidean complex.

Let K_i and L_i ($i = 0, 1, 2, \cdots$) denote the $(i + 1)$-th proper barycentric subdivisions of K_{-1} and L_{-1}, respectively. Let N_i, $i \geqq 0$, be the simplicial

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