THE CHARACTERIZATION OF FLAT PROJECTIVE SPACES BY THEIR DIFFERENTIAL INVARIANTS

By Robert Singleton

Introduction. For the ordinary projective space, Veblen has derived a flat projective connection[1, 2] and certain differential conditions satisfied by this connection. He has shown that these differential conditions are sufficient to characterize locally the ordinary projective space. T. Y. Thomas, in lectures at Princeton University in the fall of 1937, proved that the connection could be defined over a space homeomorphic to the ordinary projective space. In this paper it is shown that the ordinary projective space is completely characterized by the above differential conditions on the projective connection. This is done by finding a set of projective coördinate systems extending to the entire space. Difficulty is encountered in extending the coördinate systems uniquely since the space is not simply connected. The only other point of possible difficulty comes in showing that the coördinate systems assume each arithmetic value only once as they cover the space.

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1. Consider an n-dimensional space, P_n, which is homeomorphic to the straight lines through the origin in an $(n + 1)$-dimensional number space, A_{n+1}. P_n is coördinated with spherical coördinate systems such that the transformations between coördinate systems in the overlapping regions are analytic; and a projective connection Π is defined on P_n with components $\Pi^\alpha_{\beta\gamma}$ [Greek indices range from 0 to n, Latin from 1 to n] which are analytic functions of the coordinates x^i of the space, and transform mechanically like an affine connection. That is, for the transformation $x^i = x^i(\bar{x})$,

$$\bar{\Pi}^\alpha_{\beta\gamma} \frac{\partial x^\lambda}{\partial \bar{x}^\alpha} = \Pi^\lambda_{\mu\nu} \frac{\partial x^\mu}{\partial \bar{x}^\beta} \frac{\partial x^\nu}{\partial \bar{x}^\gamma} + \frac{\partial^2 x^\lambda}{\partial \bar{x}^\beta \partial \bar{x}^\gamma},$$

where the derivatives which include an x^0 are defined by

$$\frac{\partial x^\lambda}{\partial \bar{x}^0} = \delta^\lambda_0, \qquad \frac{\partial x^0}{\partial \bar{x}^i} = \frac{1}{\rho(\bar{x})} \frac{\partial \rho(\bar{x})}{\partial \bar{x}^i},$$

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[1] O. Veblen, *Projektive Relativitätstheorie*, Ergebnisse der Mathematik, vol. 2, 1933.

[2] O. Veblen, *Generalized projective geometry*, Journal of the London Mathematical Society, vol. 4(1929), p. 140.