A REMARK ON WIENER'S GENERAL TAUBERIAN THEOREM

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1. The following theorem[1] is due to Wiener.

THEOREM A. *Hypothesis:*[2]

(a) $k(x)$, $k^*(x)$ *belong to* $L(-\infty, \infty)$,

(b) $\int k(x)dx = \int k^*(x)dx = 1$,

(c) $| s(x) | \leqq C$,

(d) $K(x) = \int e^{-ivx}k(y)\,dy \neq 0 \qquad (-\infty < x < \infty)$,

(e) $\lim_{x\to\infty} \int k(x-y)s(y)\,dy = A$.

Conclusion:

$$\lim_{x\to\infty} \int k^*(x-y)s(y)\,dy = A.$$

Our object here is to determine extra conditions on $k(x)$ and $k^*(x)$ under which condition (c) may be replaced by the one-sided condition

$$s(x) \geqq -C.$$

(To show that Theorem A fails with this weaker condition, we take $s(x) = e^{\sigma x}$ $(\sigma > 0)$ and suppose that $k(x)$, $k(x)e^{-\sigma x}$ belong to L and $\int k(x)e^{-\sigma x}dx = 0$.) The result may be stated as follows.

THEOREM. *Hypothesis*:

(a) $k(x) \geqq 0$, $k(x)$ *belongs to* $L(-\infty, \infty)$, $\int k(x)dx = 1$,

(b) $k^*(x)$ *is continuous almost everywhere and*

$$\sum_{n=-\infty}^{\infty} \overline{\text{bd}}_{n \leqq x < n+1} | k^*(x) | < \infty, \qquad \int k^*(x)\,dx = 1,$$

(c) $s(x) \geqq -C$,

(d) $K(x) = \int e^{-ivx}k(y)dy \neq 0 \qquad (-\infty < x < \infty)$,

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[1] N. Wiener, *Tauberian theorems*, Annals of Mathematics, vol. 33(1932), pp. 1–100. The theorem stated here is Wiener's Theorem VIII.

[2] Integrals in which limits are not specified are over the range $(-\infty, \infty)$.