ON ABSOLUTELY CONVERGENT FOURIER-STIELTJES TRANSFORMS

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(1.1) **Introduction.** Suppose that $f(x)$ is of bounded variation in $(-\infty, \infty)$ and let

$$F(x) = \int e^{-iyx}\, df(y).$$

(Here, and in what follows, integrals in which the limits are not specified are always taken over the range $(-\infty, \infty)$.)

We define $\mathfrak{A}$ to be the class of functions $F(x)$ which can be expressed in this form. If $F(x)$ is periodic, it belongs to $\mathfrak{A}$ only if its Fourier series is absolutely convergent, and Wiener[1] has shown that the condition $F(x) \neq 0$, or the equivalent condition

$$\underline{\mathrm{Bd}}\, | F(x) | > 0, \tag{1.1.1}$$

is sufficient to make $[F(x)]^{-1}$ belong to $\mathfrak{A}$. The same result for almost periodic functions has been proved by Cameron[2] and Pitt.[3]

Our object here is to investigate how far this is true in the general case.

We can write

$$f(x) = h(x) + g(x) + s(x),$$

where $h(x)$ is a step function, $g(x)$ is absolutely continuous and $s(x)$ is singular in the sense defined by Lebesgue; that is, it is continuous, of bounded variation, not constant, and has derivative zero at almost all points. We can write

$$F(x) = H(x) + G(x) + S(x),$$

where $H(x)$, $G(x)$, $S(x)$ are the transforms of $h(x)$, $g(x)$, $s(x)$; and we use the letters H, G, S in this sense throughout the paper. If $F(x)$ belongs to $\mathfrak{A}$ and

$$F(x) = \int e^{-iyx}\, df(y),$$

we write

$$T\{F(x)\} = \int | df(y) |.$$

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[1] N. Wiener, *Tauberian theorems*, Annals of Mathematics, vol. 33(1932), pp. 1–100.

[2] R. H. Cameron, *Analytic functions of absolutely convergent generalized trigonometric sums*, this Journal, vol. 3(1937), pp. 682–688.

[3] H. R. Pitt, *A theorem on absolutely convergent trigonometrical series*, Journal of Math. and Phys., M. I. T., vol. 16(1938), pp. 191–195.