TAYLOR'S SERIES OF FUNCTIONS OF SMOOTH GROWTH IN THE UNIT CIRCLE

BY W. T. MARTIN AND N. WIENER

1. **Introduction.** We have recently proved a series of results connecting the growth of an entire function with the growth of certain expressions depending on the coefficients.[1] There is a closely related series of results which concern, not the growth of an entire function, but the growth of a function as it approaches the circle of convergence. As will be seen in the formulation of our results, there is a precise methodological similarity between theorems of the two types, and, indeed, they differ merely by the fact that a parameter which is negative in one case is positive in the other. It is nevertheless worth while to give an explicit formulation of the theorems because they are of less familiar character than the other set of theorems and involve results which have only recently been obtained by Vijayaraghavan and Wiener, Avakumović and Karamata, and Pitt (see footnotes 4, 6) in contrast with the theorems of the entire type which are best exemplified by the classical theory of the Borel summation. The circle of convergence theorems are more directly applicable to a series of interesting problems in the analytical theory of numbers, and, in particular, they allow us to carry Tauberian methods in the problem of partitions much further than Hardy and Ramanujan at one time believed possible.[2] There is a considerable prospect of their further utility in unexplored portions of this field. It is true that the revolutionary work of Rademacher[3] overshadows all less perfect methods, whether Tauberian or not. We wish to call attention to the fact that it is possible by an artifice to eliminate the condition of positivity which is necessary for Tauberian theorems of the type considered, or, more accurately, to guarantee its satisfaction by a consideration not of a singularity of a power series at an individual point on the circle of convergence, but rather the behavior of the integral of the square of its modulus as we approach the circle of convergence.

There will be many places in which the detail of proof is so similar to that of our previous paper that anything like completeness is scarcely necessary. Where this is the case we shall consider ourselves at liberty to present our argument in a highly schematic form and to refer to our previous paper for technical details.

Received January 12, 1938.

[1] N. Wiener and W. T. Martin, *Taylor's series of entire functions of smooth growth*, this Journal, vol. 3(1937), pp. 213–223.

[2] G. H. Hardy and S. Ramanujan, *Asymptotic formulae in combinatory analysis*, Proc. of the London Math. Soc., vol. 17(1918), pp. 75–115. See especially pp. 89, 90.

[3] H. Rademacher, *On the partition function* $p(n)$, Proc. of the London Math. Soc., vol. **43** (1937), pp. 241–254.