A DETERMINATION OF THE AUTOMORPHISMS OF CERTAIN ALGEBRAIC FIELDS

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1. **Introduction.** The problem of determining all equations with rational coefficients having a given Galois group has been solved only in a few cases. All normal cubic and quartic equations having a prescribed group have been explicitly determined [8][1] as equations whose coefficients are rational in certain parameters. The possibility of so expressing all equations of higher degree having a prescribed group was discussed by E. Noether [7]. Less explicit determinations of normal equations of degrees five [5] and eight [4] have been made.

The companion problem of determining explicitly the automorphisms of a given normal field in terms of the coefficients of the defining equation has met with less success. It has been solved for the cyclic cubic [9]. Generating automorphisms for cyclic quartic and octic fields were found by Albert [1] and for cyclic quintic fields by Hull [5], but not explicitly in terms of the coefficients.

In the present paper, all automorphisms are explicitly obtained by purely algebraic methods for the cyclic cubic, quartic, and sextic, the quartic with the four-group, the sextic with the symmetric group, and the octics with the Abelian groups of types (2, 2, 2) and (2, 4). The determination of the parametric representations of the most general equations defining these fields was an integral part of the determination of the automorphisms, and while for $n = 3$ and $n = 4$ these results merely confirm known facts, the purely rational method of their attainment should not be without interest.

The computation in the cases $n = 6$ and $n = 8$ was brought within practicable limits by a free use of matric theory; in particular, of a theorem of Williamson [10]. This was used with particular success in the concluding section to obtain from the results of Albert a one-parameter family of cyclic octics over the rational field together with their automorphisms.

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2. **Cyclic cubics.** If the reduced cubic

$$f(x) = x^3 + px + q = 0 \tag{1}$$

is normal over the rational field F, it has the roots

$$\alpha, \qquad \alpha' = \theta_1(\alpha), \qquad \alpha'' = \theta_2(\alpha),$$

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[1] The numbers in brackets refer to the bibliography at the end.