

QUATERNARY CREMONA GROUPS OF TERNARY TYPE

BY FRANK C. GENTRY

Introduction. We consider the possibility of using involutions determined by webs of quartic surfaces of degree 2 as generators of groups of Cremona transformations in space. Coble¹ has discussed the same problem using involutions determined by webs of cubic surfaces as generators.

For a web of quartic surfaces of degree 2 to contain in its base a curve of index numbers² (α'_0, α'_1) and of multiplicity i , a simple curve (α_0, α_1) meeting the multiple curve s times, B_j j -fold points ($j = 1, 2, 3, \dots$), Hudson³ gives for the postulation P and the equivalence E the formulas:

$$P = \frac{i(i+1)}{12} \{36\alpha'_0 + (2i+1)\alpha'_1\} + 6\alpha_0 + \frac{\alpha_1}{2} - is$$

$$+ \sum_j \frac{j(j+1)(j+2)B_j}{6} = 31,$$

$$E = i^2(12\alpha'_0 + i\alpha'_1) + 12\alpha_0 + \alpha_1 - (3i-1)s + \sum_j j^3 B_j = 62.$$

The following solutions of these equations, for $i > 1$, $\alpha'_0 \neq 0$, $B_1 \neq 0$, lead to webs of non-degenerate quartic surfaces:

No.	i	α'_0	α'_1	α_0	α_1	s	B_1	B_2	No.	i	α'_0	α'_1	α_0	α_1	s	B_1	B_2
I	2	1	-2	6	-18	5	1	0	VIII	2	1	-2	2	-4	2	4	2
II	2	1	-2	7	-30	5	1	0	IX	2	1	-2	0	0	0	6	3
III	2	1	-2	5	-12	4	2	0	X	2	2	-6	4	-16	4	2	0
IV	2	1	-2	6	-24	4	2	0	XI	2	2	-6	3	-10	3	3	0
V	2	1	-2	5	-18	3	3	0	XII	2	2	-6	2	-4	2	4	0
VI	2	1	-2	4	-12	2	4	0	XIII	2	2	-6	0	0	0	6	1.
VII	2	1	-2	4	-8	4	2	1									

Sharpe and Snyder⁴ have determined the homaloidal webs and fundamental and principal elements of the involutions of Cases II, IV, V and VI. Except in

Received October 18, 1937.

¹ A. B. Coble, *Groups of Cremona transformations in space of planar type*, I and II, this Journal, vol. 2 (1936), pp. 1, 205.

² A. B. Coble, *Restricted systems of equations*, I, II, American Journal of Mathematics, vol. 36 (1914), pp. 167, 295.

³ Hilda P. Hudson, *Cremona Transformations in Plane and Space*, Chapter XI, Cambridge, 1927.

⁴ F. R. Sharpe and V. Snyder, *Certain types of involutorial space transformations*, Transactions of the American Mathematical Society, vol. 21 (1920), pp. 52-78.