CERTAIN INTEGRAL FUNCTIONS RELATED TO EXPONENTIAL SUMS

By Harry Matison

Introduction. The study of Borel transforms by several mathematicians, in particular by Pólya,[1] has brought out many interesting relations between the singularities of a function $f(z)$ defined by a power series

$$f(z) = \sum_{n=0}^{\infty} a_n z^n$$

on the one hand, and the rate of growth and distribution of zeros of its Borel transform

$$F(z) = \sum_{n=0}^{\infty} \frac{a_n}{n!} z^n \tag{1}$$

on the other. As the prototype in these considerations, there is the case in which $f(z)$ is a rational function with simple poles

$$f(z) = \frac{\alpha_1}{1 - \beta_1 z} + \cdots + \frac{\alpha_k}{1 - \beta_k z},$$

and the Borel transform is a sum of exponentials

$$F(z) = \alpha_1 e^{\beta_1 z} + \cdots + \alpha_k e^{\beta_k z}.$$

This case has been studied in detail by Pólya and Schwengeler.[2] It is easily seen that here the coefficients a_n can be interpolated by the function

$$p(t) = \alpha_1 \beta_1^t + \cdots + \alpha_k \beta_k^t$$

with $a_n = p(n)$. In the particular case in which the β_ν all lie on the unit circle, $p(t)$ becomes a special type of almost-periodic function. The generalization to general uniformly almost-periodic functions (u. a. p. functions) has been studied by Bochner and Bohnenblust[3] as regards $f(z)$. We propose to make a

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[1] G. Pólya, *Untersuchungen über Lücken und Singularitäten von Potenzreihen*, Math. Zeitschrift, vol. 29 (1929), p. 549.

[2] Pólya, *Geometrisches über die Verteilung der Nullstellen gewisser ganzer transzendenter Funktionen*, Münchner Berichte, 1920.

E. Schwengeler, Dissertation, Zürich, 1923.

[3] S. Bochner and F. Bohnenblust, *Analytic functions with almost periodic coefficients*, Annals of Math., vol. 35 (1934), p. 152.