AN ANALOGUE OF THE VON STAUDT-CLAUSEN THEOREM

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1. **Introduction.** Let $GF(p^n)$ denote a fixed Galois field, and x an indeterminate over the field. The function[1]

$$\psi = \psi(t) = \sum_{i=0}^{\infty} \frac{(-1)^i}{F_i} t^{p^{ni}}, \tag{1.1}$$

where

$$[i] = x^{p^{ni}} - x, \qquad F_i = [i][i-1]^{p^n} \cdots [1]^{p^{n(i-1)}}, \qquad F_0 = 1, \tag{1.2}$$

is closely connected with the arithmetic of polynomials in the $GF(p^n)$. In this paper we study the coefficients in the reciprocal of (1.1), more precisely in t/ψ. In particular we shall be interested in proving an analogue of the von Staudt-Clausen theorem for these coefficients.

In order to define properly the coefficients in the reciprocal it is necessary to define a "normalizing" factor (analogous to $n!$ in ordinary arithmetic). This is done in the following way. Let

$$m = \alpha_0 + \alpha_1 p^n + \cdots + \alpha_s p^{ns} \qquad (0 \leqq \alpha_j < p^n)$$

be the canonical expansion of m to the base p^n; then we put

$$g(m) = F_0^{\alpha_0} F_1^{\alpha_1} \cdots F_s^{\alpha_s}, \qquad g(0) = 1, \tag{1.3}$$

where F_i has the same significance as in (1.2). Thus for example

$$g(p^{ns}) = F_s, \qquad g(p^{ns} - 1) = (F_1 \cdots F_{s-1})^{p^n - 1}.$$

We may now define the coefficients of the reciprocal by means of

$$\frac{t}{\psi} = \sum_{m=0}^{\infty} \frac{B_m}{g(m)} t^m, \tag{1.4}$$

the summation obviously containing only terms in which m is a multiple of $p^n - 1$. Clearly $B_0 = 1$ and B_m is a rational function of x. The analogy between B_m and the ordinary Bernoulli numbers is brought out by the relation[2]

$$\sum \frac{1}{E^m} = \frac{B_m}{g(m)} \xi^m \qquad (p^n - 1 | m),$$

where the summation is over *all* primary polynomials E, and

$$\xi = \lim_{k=\infty} \frac{[1]^{p^{nk}/(p^n-1)}}{[k][k-1] \cdots [1]}.$$

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[1] See this Journal, vol. 1 (1935), pp. 137–168. This paper will be cited as DJ.

[2] DJ, p. 161, Theorem 9.3.