TAYLOR'S SERIES OF ENTIRE FUNCTIONS OF SMOOTH GROWTH

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1. **Introduction.**[1] Let $a_n \geqq 0$ $(n = 1, 2, \cdots)$, and

$$\sum_1^\infty a_n x^n \sim sH(x) \qquad (x \to \infty). \tag{1}$$

We may rewrite (1) in the form

$$\sum_1^\infty a_n e^{n\xi - F(\xi)} \to s \qquad (\xi \to \infty), \tag{2}$$

where

$$H(x) = e^{F(\log x)} \tag{3}$$

We shall derive the following Tauberian theorem.

Theorem 1. *Let* $a_n \geqq 0$ $(n = 1, 2, \cdots)$, *and let* (2) *hold, where*

(4) *F is four times continuously differentiable for* $a \leqq x$ *for some* a;

$$F''(x) \geqq \text{const.} > 0 \textit{ for } a \leqq x; \tag{5}$$

$$F'''(x) = o([F''(x)]^{\frac{3}{2}}) \qquad (x \to \infty); \tag{6}$$

$$F^{\text{iv}}(x) = o([F''(x)]^2) \qquad (x \to \infty). \tag{7}$$

Then for any positive λ,

$$\lim_{x \to \infty} \frac{(2\pi)^{\frac{1}{2}}}{\lambda} \sum_{x \leqq \psi(n) \leqq x+\lambda} a_n e^{nG(n) - F(G(n))} = s, \tag{8}$$

where $\psi(x)$ *is defined by*

$$\int_a^x [G'(x)]^{\frac{1}{2}} dx = \psi(x), \tag{9}$$

and $G(x)$ *is the inverse function to* $F'(x)$ *for* $a \leqq x$.

As a converse to this theorem we shall prove

Theorem 2. *Let* $a_n \geqq 0$ $(n = 1, 2, \cdots)$, *and let* (8) *hold for every positive value of* λ, *where F fulfills the conditions* (4), (5), (6) *and* (7) *and* ψ *and* G *are defined as in Theorem* 1. *Then* (2) *holds.*

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[1] The authors' attention was directed to problems of this type by Professor Vijayaraghavan. It has come to our attention that a similar group of problems has been recently attacked by Mr. Kales of Brown University by quite different methods. While neither direction of work is reducible to the other, Mr. Kales' priority in entering the field is clear.