NON-SEPARATING TRANSFORMATIONS

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1. Introduction. If A is a compact continuum and $T(A) = B$ is a single-valued continuous transformation, then T will be said to be *non-separating* provided that no set $T^{-1}(b)$, $b \in B$, separates A. It is obvious that any non-separating transformation is non-alternating.[1] However, it can easily be seen by simple examples that not every non-alternating transformation is non-separating; not every non-separating transformation is monotone;[2] and not every monotone transformation is non-separating.

Since any continuous transformation between two compact metric spaces A and B is equivalent[3] to an upper semi-continuous decomposition[4] of A into disjoint closed sets where the hyperspace of the decomposition is homeomorphic with B, any non-separating transformation $T(A) = B$ is equivalent to an upper semi-continuous decomposition of A into sets which do not separate A.

All transformations used in this paper will be assumed to be single-valued and continuous.

2. Some characteristic properties.

THEOREM 2.1. *If A and B are compact continua, a necessary and sufficient condition in order that $T(A) = B$ be non-separating is that T be non-alternating and B contain no cut points.*

Proof. To prove the necessity, in view of our remarks in the above section, we need only show that B can contain no cut points. If, for some point b of B, there were a separation $B - b = B_1 + B_2$, then $T^{-1}(b)$ would separate A into the two mutually separated sets $T^{-1}(B_1)$ and $T^{-1}(B_2)$ because of the continuity of T. The sufficiency follows at once from a theorem of G. T. Whyburn's[5] which states that if B is connected and $T(A) = B$ is non-alternating, then a point x of B is a cut point of B if and only if $T^{-1}(x)$ separates A.

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[1] A continuous transformation $T(A) = B$ is *non-alternating* provided that for any $x, y \in B$, $T^{-1}(x)$ does not separate $T^{-1}(y)$ in A. See G. T. Whyburn, American Journal of Mathematics, vol. 56 (1934), no. 2, pp. 294–302.

[2] A continuous transformation $T(A) = B$ is *monotone* provided that each set $T^{-1}(b)$, $b \in B$, is connected. See C. B. Morrey, Jr., American Journal of Mathematics, vol. 57 (1935), pp. 17–50, and G. T. Whyburn, loc. cit.

[3] C. Kuratowski, Fundamenta Mathematicae, vol. 11 (1928), pp. 169–185.

[4] R. L. Moore, Transactions of the American Mathematical Society, vol. 27 (1925), pp. 416–428.

[5] See p. 295 of his paper *Non-alternating transformations*, loc. cit.