TWO SYSTEMS OF POLYNOMIALS FOR THE SOLUTION OF LAPLACE'S INTEGRAL EQUATION

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1. In the radiation and conduction problems[1] in which the integral equation

$$f(x) = \int_0^\infty e^{-xt} g(t) dt$$

occurs, the variable x takes positive values, and so the function $g(t)$ is to be derived from the values of x for $x > 0$. In the inversion formulas given by Lord Kelvin[2]

$$f(x) = x^{-\frac{1}{2}} \int_0^\infty \begin{matrix}\cos\\ \sin\end{matrix} (u/2x) F(u) du,$$

$$g(t) = (\pi^3 t)^{-\frac{1}{2}} \int_0^\infty du \int_0^\infty \begin{matrix}\mathrm{ch}\\ \mathrm{sh}\end{matrix} (ut)^{\frac{1}{2}} \begin{matrix}\cos\\ \sin\end{matrix} (ut)^{\frac{1}{2}} \begin{matrix}\cos\\ \sin\end{matrix} (u/2x) x^{-\frac{3}{2}} f(x) dx,$$

the integration with respect to x does indeed run from 0 to ∞, but conditions to be satisfied by $f(x)$ or $F(u)$ sufficient to make one of these formulas valid have not yet been formulated in a useful form. A similar remark applies to the somewhat analogous formula of F. Sbrana.[3] A more complete inversion formula in which the integration runs from $x = 0$ to $x = \infty$ has been given recently by R. E. A. C. Paley and N. Wiener.[4] In Murphy's first method[5] of solving the integral equation, $xf(x)$ is expanded in a series of ascending powers of x^{-1} and $g(t)$ is expressed as the coefficient of x^{-1} in $f(x)e^{xt}$ which, by Cauchy's theory, may be expressed as a contour integral. This method was generalized by Lerch[6] for the case in which $x^\nu f(x)$ can be expanded in a series of powers of x^{-1} and the resulting expression can be transformed into a contour integral resembling that used in the well-known inversion formula of Laplace, Riemann and Mellin.

Murphy also gave a method in which $f(x)$ is expanded in a series of inverse

Received February 3, 1936; presented to the American Mathematical Society, November 30, 1935.

[1] H. Poincaré, Jour. de Phys., vol. 11 (1912), p. 34; L. Silberstein, Phil. Mag., vol. 15 (1932), p. 375; H. Bateman, Proc. Camb. Phil. Soc., vol. 15 (1910), pp. 423–427.

[2] Lord Kelvin, Camb. Math. Jour., vol. 3 (1842), p. 170; Math. and Phys. papers, vol. 1, p. 10. See also H. Bateman, Messenger of Math., vol. 57 (1928), p. 145.

[3] F. Sbrana, Rend. Lincei, (5), vol. 31 (1922), pp. 454–456.

[4] *Fourier Transforms in the Complex Domain*, Chapter 3.

[5] R. Murphy, Camb. Phil. Trans., vol. 4 (1833), p. 353.

[6] M. Lerch, Rozpravy, vol. 2 (1893), p. 9; Fortschritte der Math., vol. 25 (1893), p. 482.