STUDIES IN THE SUMMABILITY OF FOURIER SERIES BY NÖRLUND MEANS

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I. Preliminary remarks and formulas

1. **Introduction.** In a paper published in 1932, E. Hille and J. D. Tamarkin [3][1] discussed the application of Nörlund means to the summation of Fourier series and certain associated series. They gave conditions for the effectiveness of the method in certain senses.

It is our purpose to consider further effectiveness problems of the theory for this method of summability. In particular, we shall consider its effectiveness for the summation of the Fourier series and conjugate Fourier series at points of "(C, α) continuity", and for the summation of the r-th derived series of both of these series. We shall also consider the strong summability of the two series when the partial sums are replaced by their Nörlund transforms.

2. **Nörlund means.** For a sequence $\{x_n\}$, the generalized Nörlund limit (if it exists) is defined as

$$(2.01)\qquad (N, p_\nu)\text{-}\lim x_n = \lim_{n\to\infty} P_n^{-1}(p_n x_0 + p_{n-1}x_1 + \cdots + p_0 x_n),$$

where $\{p_\nu\}$ is a sequence of complex numbers such that $P_n \equiv p_0 + p_1 + \cdots + p_n \neq 0$. The conditions of regularity are

$$(2.02)\qquad \sum_{k=0}^{n} |p_k| < C|P_n|, \qquad p_n/P_n \to 0,$$

where C is a fixed positive constant.

N. E. Nörlund [8] proved some properties of these means assuming $p_n > 0$ and $p_n/P_n \to 0$. Such a definition of limitation, however, had already been given by G. F. Woronoi [17], who assumed that $p_n > 0$ and that $n^{-\alpha}P_n$ is bounded for some value of α. We shall use the symbol (N, p_ν) to denote the Nörlund method of summation defined by the sequence $\{p_\nu\}$. If

$$p_n = \binom{\epsilon + n - 1}{n}, \qquad P_n = \binom{\epsilon + n}{n},$$

the corresponding method (N, p_ν) reduces to the Cesàro method (C, ϵ).

3. **Notation.** We shall consider functions $f(x)$ integrable in the sense of Lebesgue and periodic of period 2π. If

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt\,dt, \qquad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin nt\,dt,$$

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[1] Numbers in square brackets refer to the bibliography at the end.