THE SIMPLE GROUP OF ORDER 25920

By J. S. Frame

1. Among the 53 known simple groups of composite order less than on million,[1] 42 may be represented as linear fractional modular groups on two o three variables. There are in addition three other alternating groups, and three multiply transitive groups. Of the five remaining groups, three are hyperorthogonal groups on three variables, whose irreducible representations were discussed in a recent paper.[2] The other two, of orders 25920 and 979200 respectively, may be defined as the "abelian linear groups"[1] $A(4, 3)$ and $A(4, 4)$. The first of these is also isomorphic to the hyperorthogonal group $HO(4, 4)$ on four variables in a modular field of four marks. Thus it is the smallest example both of the "abelian linear group" and of the hyperorthogonal group on more than three variables. We are interested here in its properties from the latter point of view, and shall obtain the complete table of characters of its irreducible representations.

The group $HO(m, q^2)$ may be defined as the quotient group with respect to its central of the special unitary group G_m, consisting of matrices of degree m and determinant 1, with coefficients from a finite field $GF(q^2)$ of q^2 marks. Here $q = p^s$ is a power of the prime p, and "conjugate imaginaries" are defined by the equation $\bar{x} = x^q$. We may think of the transformations of the group G_m as operating on a set of vectors in an m-dimensional space where the coördinates are marks of the $GF(q^2)$. All multiples of a given vector will be said to form a *ray*. The inner product, $(a \mid b) \equiv \sum_{i=1}^{m} \bar{a}_i b_i$, of two vectors a and b is invariant under each transformation of the unitary group G_m, so that the *isotropic* vectors—those for which $(a \mid a) = 0$—are permuted among themselves, and so are the remaining *non-isotropic* vectors, for which $(a \mid a) \neq 0$. It has been shown[3] that the permutation groups thus induced on the rays of each of the two types are transitive. If a single vector be selected from each ray, these vectors undergo a *monomial substitution* under the group G_m, with multipliers which, although appearing as marks from the $GF(q^2)$, may be replaced by suitably chosen $(q^2 - 1)$-th roots of unity from the field of complex numbers. It has also been shown that for $m = 3$ the permutation group on the isotropic rays is doubly transitive, and the corresponding monomial groups either have just two irreducible components, or are irreducible. In this way more than

Received February 20, 1936.

[1] L. E. Dickson, *Linear Groups*, 1901, p. 309.

[2] J. S. Frame, *Some irreducible monomial representations of hyperorthogonal groups*, this Journal, vol. 1 (1935), p. 442.

[3] J. S. Frame, *Unitäre Matrizen in Galoisfeldern*, Commentarii Mathematici Helvetici, vol. 7 (1935), p. 97.