TEMPERATURE DISTRIBUTION IN A SLAB OF TWO LAYERS

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The list of solved problems in one-dimensional heat conduction in composite walls does not seem to include the cases in which the initial temperature distribution is arbitrary. The case of the semi-infinite composite solid with an arbitrary initial temperature distribution has been treated recently by Lowan.[1] The solution of the corresponding problem for the wall of finite thickness seems desirable for the sake of completeness. It is solved here by application of the Laplace transformation.

The problem under consideration is that of finding the one-dimensional distribution of temperature in a slab consisting of two layers of different materials whose outer parallel faces are held at fixed temperatures, when the initial temperature distribution in each layer is arbitrarily given. Let the thickness of the layers be a, b, and let $x = 0$ be taken as the surface of separation. Then the boundary conditions on the temperatures $T_1(x, t)$, $T_2(x, t)$ in the two layers may be written

$$\begin{aligned} &T_1(-a, t) = 0, \qquad \lim_{t \to 0} T_1(x, t) = f(x), \qquad -a < x < 0, \\ &T_2(b, t) = c, \qquad \lim_{t \to 0} T_2(x, t) = g(x), \qquad 0 < x < b, \end{aligned} \tag{1}$$

$$T_1(0, t) = T_2(0,t), \qquad K_1 \frac{\partial}{\partial x} T_1(x, t) = K_2 \frac{\partial}{\partial x} T_2(x, t), \qquad x = 0, \tag{2}$$

where K_1, K_2 are the thermal conductivities of the two layers.

It is easily seen that the temperatures T_1 and T_2 can be obtained by the composition of known temperature formulas and a simpler unknown formula. Let each of the three pairs of temperature functions u_1, u_2, v_1, v_2 and w_1, w_2 satisfy the conditions (2), and let

$$\begin{aligned} &u_1(-a, t) = 0, \qquad v_1(-a, t) = 0, \qquad w_1(-a, t) = 0, \\ &u_2(b, t) = c, \qquad v_2(b, t) = 0, \qquad w_2(b, t) = 0, \\ &\lim_{t \to 0} u_1(x, t) = 0, \qquad \lim_{t \to 0} v_1(x, t) = f(x), \qquad \lim_{t \to 0} w_1(x, t) = 0, \qquad -a < x < 0, \\ &\lim_{t \to 0} u_2(x, t) = 0, \qquad \lim_{t \to 0} v_2(x, t) = 0, \qquad \lim_{t \to 0} w_2(x, t) = g(x), \qquad 0 < x < b. \end{aligned}$$

Then the temperature functions

$$T_1 = u_1 + v_1 + w_1, \qquad T_2 = u_2 + v_2 + w_2$$

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[1] A. N. Lowan, *Heat conduction in a semi-infinite solid of two different materials*, this Journal, vol. 1 (1935), pp. 94–102.