# TRANSFORMATIONS OF MULTIPLE SEQUENCES

BY HUGH J. HAMILTON

## §1. Introduction and definition of notation

**1.1. Notation.** In order to treat $n$-tuple sequences with any degree of facility, it is necessary to introduce an abbreviated notation. The present paper uses one defined as follows.

The single letter $m$ will denote an ordered set of $n$ positive, integral variables, and $k$ another such, homologous to $m$. A fixed value-set for $m$ will be denoted by $r$, and the $i$-th of an infinite sequence of such sets by $m_i$. The symbols $p$ and $k_i$ are to be interpreted in an analogous sense with respect to $k$.

Generic representation for conjugate, proper, ordered subsets of any of these sets is to be obtained by affixing the superscripts 1 and 2, respectively, to the symbol denoting the set, and further subsets of like character with respect to either of these are to be represented by adjoining to the present superscript further numbers 1 and 2, respectively, etc. Two sets whose symbolic representations involve the same superscript are to be considered homologous. When the implication of this homology is *not* intended, the numbers 1 or 2 are replaced by 3 or 4, respectively, in one of the symbols. Thus $k^3$, $k^4$ are conjugate, but homologously independent of $m^1$, $m^2$.

A single element of $k$ will be denoted generically by $\kappa$ (or $\lambda$), and a fixed value of it by $\pi$. The corresponding element of $m$ will be represented by $\mu$.

All other letters are to be interpreted in the customary sense.

By relations like $k^{12} = p^{12}$ or $m_i > m_{i-1}$ are to be understood all sets of relations of the same form between corresponding elements of the two sets. In particular, the equation $k = p(m)$ is equivalent to the set of $n$ equations $\kappa = \pi(m)$. The notations $k^3 = 1, 2, \cdots$ or $m^1 \geqq M$ imply the corresponding range of variation for each separate element of the set. However, inequalities like $k \not\leqq M$ mean simply that *not every element* in the set is less than or equal to $M$.

Except when, by the nature of the situation, such would obviously be absurd, all relations involving subsets of $k$ or of $m$ are to be understood as implying the set of such relations for all possible choices of such subsets (with respect to position and, except when the subset consists only of $\kappa$ or $\mu$, with respect to dimension).

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