ON LOCALLY CONNECTED SPACES

By R. L. Wilder

In a recent paper,[1] for the purpose of characterizing the boundaries of uniformly locally i-connected domains in n-space, I have designated a certain class of compact metric spaces as "generalized closed n-manifolds" (= g. c. n-m.). Considered as boundaries of domains in euclidean spaces, they form the exact analogues, for higher dimensions, of the domain boundaries in 3-space represented by the class of all closed 2-dimensional orientable manifolds; in particular, their Betti groups are finite and they satisfy the Poincaré duality, and, moreover, have the same sort of relations (as regards linkings, dualities, etc.) to their complements as have the ordinary 2-manifolds to their boundaries in 3-space; in 3-space they are identically the ordinary 2-manifolds. Among the problems so far not treated is that of proving the finiteness of the Betti numbers of the g. c. n-m. when considered as an abstract space, not necessarily imbedded in euclidean space. It was in considering this problem that the results of the present paper were obtained.

From the properties of the abstract g. c. n-m. it follows that such a space is locally i-connected, in the sense of Vietoris cycles (= V-cycles),[2] for $0 \leqq i \leqq n-2$.

Received May 22, 1935.

[1] *Generalized closed manifolds in n-space*, Annals of Mathematics, vol. 35 (1934), pp. 876–903. We refer to this paper hereafter as G. C. M.

[2] Shortly after this paper was presented to the American Mathematical Society (see its Bulletin, vol. 41 (1935), p. 202, abstract no. 178), there appeared two papers intimately related, in part, to the subject matter of the present paper. We refer to the following: K. Borsuk, *Un théorème sur les groupes de Betti des ensembles localement connexes en toutes les dimensions* $\leqq n$, Fundamenta Mathematicae, vol. 24 (1935), pp. 311–316; S. Lefschetz, *Chain-deformations in topology*, this Journal, vol. 1 (1935), pp. 1–18. In the former article, the finiteness of the Betti groups is established for spaces locally connected in the sense of Lefschetz's *Topology* (p. 91); in the latter, a similar result is obtained using a more general type of local connectedness. The reason for our use of Vietoris cycles, which we term V-cycles hereafter, in the local connectedness is due to the origin of our problem in the g. c. n-m., whose origin in turn in the study of domain boundaries necessitated a formulation of their properties in terms of such cycles. As a point of departure, it seems to us that the local connectedness in the sense of Lefschetz may be preferable in that it avoids certain complications of proof encountered in using the "infinite" cycles; thus, in the present work we have had to restrict our chains to those obtainable using a finite coefficient ring, for reasons of convergence, whereas in using singular chains the type of coefficient ring used is not material. Apparently, however, local connectedness over a range of dimensions $0 \leqq i \leqq k$ is more general in the sense of the V-cycles than in the sense of either singular spheres or cycles, since the two latter types imply the former but not conversely. For instance, we may construct a space consisting of an infinite set of mutually exclusive "Poincaré spaces" converging to a point and successively connected by simple arcs which