# GROUPS INVOLVING FIVE COMPLETE SETS OF NON-INVARIANT CONJUGATE OPERATORS

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1. **Introduction.** The finite abstract groups involving no more than five complete sets of conjugate operators were determined by G. A. Miller.[1] W. Burnside[2] published the same results in his *Theory of Groups of Finite Order*. Some general theorems on the number of sets of conjugate operators in a group of finite order have been published by G. A. Miller.[3] In this paper we prove two theorems on complete sets of non-invariant conjugate operators in a group of finite order, and derive the abstract groups involving five complete sets of non-invariant conjugate operators.

2. **Non-invariant sets of conjugate operators in a finite group.** Let $G$ represent a group of finite order $g$, and let $H$, of order $h$, represent the central of $G$. Assume that $G$ is non-abelian, and hence $G$ contains $k$, $k > 0$, complete sets of non-invariant conjugate operators. From the isomorphism between $G$ and the quotient group $G/H$, we may state the

Lemma. *A necessary and sufficient condition that a group $G$ contain more than $k$, $k > 0$, complete sets of non-invariant conjugate operators, if the central quotient group contains $k$ complete sets of conjugate operators, and the identity, is that the order $h$, of $H$, exceed unity.*

We may state

Theorem 1. *A group $G$ containing $k$, $k > 0$, complete sets of non-invariant conjugate operators, with a central of order greater than unity, has a central quotient group which involves not more than $k + 1$ complete sets of conjugate operators, at least two of which are composed of invariant operators, if this number is $k + 1$.*

The first part of the theorem follows from the fact that $G$ and $G/H$ are isomorphic, and hence $G/H$ does not involve more complete sets of conjugates than $G$. Assume that $G/H$ involves $k + 1$ complete sets of conjugate operators, of which $k$ are composed of non-invariant operators. The order of $G/H$ is divisible by at least two distinct primes $p$ and $q$, since a group of order $p^m$, $p$ a prime, contains invariant operators in addition to the identity. The operators in a co-set of $G$ with respect to $H$ are all conjugate. Therefore, all of the operators in the same co-set are of the same order. This is a contradiction, since there are operators of two different orders in one of the co-sets corresponding

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[1] Archiv der Math. und Phys., vol. 17 (1910), p. 199.

[2] W. Burnside, *Theory of Groups of Finite Order*, 2nd ed., 1911, Note A.

[3] Trans. Amer. Math. Soc., vol. 20 (1919), p. 262; Amer. Journal of Math., vol. 54 (1932), p. 110.