ON THE CHARACTERISTIC EXPONENTS IN CERTAIN TYPES OF PROBLEMS OF MECHANICS

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1. **Introduction.** Some years ago H. E. Buchanan[1] published a discussion of periodic orbits near the straight line and equilateral triangle positions in the problem of three finite bodies. More recently he has discussed[2] small oscillations of the so-called neutral helium atom near the straight line and equilateral triangle positions. In all four of these problems the characteristic exponents $0, 0, \pm i\omega$, where ω is the angular velocity, occurred. This paper is an attempt to find out whether one could have predicted the appearance of these exponents from the known integrals of the equations.

All of the problems mentioned above were set up in axes rotating uniformly with angular speed ω. The differential equations in each case may be written in the form

$$\begin{aligned} &\frac{d^2\bar{\xi}_i}{dt^2} - 2\,\omega\,\frac{d\bar{\eta}_i}{dt} = \omega^2\,\bar{\xi}_i + \frac{1}{m_i}\,\frac{\partial U}{\partial \bar{\xi}_i}\,, \\ (1) \qquad &\frac{d^2\bar{\eta}_i}{dt^2} + 2\,\omega\,\frac{d\bar{\xi}_i}{dt} = \omega^2\,\bar{\eta}_i + \frac{1}{m_i}\,\frac{\partial U}{\partial \bar{\eta}_i}\,, \\ &\frac{d^2\bar{\zeta}_i}{dt^2} = \frac{1}{m_i}\,\frac{\partial U}{\partial \bar{\zeta}_i} \qquad (i = 1, 2, 3)\,. \end{aligned}$$

The function U in the three body problem is

$$\frac{m_1m_2}{r_{12}} + \frac{m_1m_3}{r_{13}} + \frac{m_2m_3}{r_{23}}\,,$$

and in the helium atom it is

$$\frac{e_1e_2}{r_{12}} - \frac{e_1e_3}{r_{13}} + \frac{e_2e_3}{r_{23}}\,,$$

e_2 being the charge on the nucleus and $-e_1$, $-e_3$ the charges on the electrons.

These equations can be thrown into the form

$$\frac{dx_i}{dt} = X_i(x)\,, \qquad (i = 1, \cdots, 18)$$

by the simple transformation

$$\bar{\xi}_i = x_i\,, \quad \bar{\xi}_i' = x_{i+3}\,, \quad \bar{\eta}_i = x_{i+6}\,, \quad \bar{\eta}_i' = x_{i+9}\,, \quad \bar{\zeta}_i = x_{i+12}\,, \quad \bar{\zeta}_i' = x_{i+15}\,.$$

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[1] Am. Journal of Math., vol. 45 (1923), pp. 93–121; and vol. 50 (1928), pp. 613–626.

[2] Am. Math. Monthly, vol. 38 (1931), pp. 511–521 and vol. 40 (1933), pp. 532–537.