LINEAR DIFFERENTIAL EQUATIONS WITH ALMOST PERIODIC COEFFICIENTS

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Introduction. In this paper we shall deal with the homogeneous system of differential equations

$$\frac{d}{dt}\xi_\mu(t) = \sum_{\nu=1}^{N} \alpha_{\mu,\nu}(t)\xi_\nu(t) \qquad (\mu = 1, \cdots, N)$$

and the non-homogeneous system

$$\frac{d}{dt}\xi_\mu(t) = \beta_\mu(t) + \sum_{\nu=1}^{N} \alpha_{\mu,\nu}(t)\xi_\nu(t) \qquad (\mu = 1, \cdots, N),$$

where the $\alpha_{\mu,\nu}(t)$, $\beta_\mu(t)$ and $\xi_\mu(t)$ are complex a.p. (almost periodic)[1] functions of the real variable t. It is the purpose of this paper to point out the manner in which the a.p. solutions of the above equation depend on the modules[2] of the $\alpha_{\mu,\nu}(t)$ and $\beta_\mu(t)$. We shall be interested in determining the form of those solutions which are a.p., and not in determining conditions under which a.p. solutions exist. Such conditions have already been given in papers by Favard,[3] Bochner,[4] and Cameron.[5]

For the sake of simplicity in notation, we rewrite the above equations in the form

$$D[x(t)] = A(t)\cdot x(t) \tag{1}$$

and

$$D[x(t)] = A(t)\cdot x(t) + b(t), \tag{2}$$

where $x(t)$ and $b(t)$ are N-dimensional vectors having the components $\xi_1(t), \cdots, \xi_N(t)$ and $\beta_1(t), \cdots, \beta_N(t)$ respectively, $A(t)$ is the matrix of the $\alpha_{\mu,\nu}(t)$, and $A\cdot x$

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[1] H. Bohr, *Zur Theorie der fastperiodischen Funktionen*, Acta Mathematica, vol. 45 (1925), pp. 29–127, esp. p. 30.

[2] A module is a set of numbers which is closed under addition and subtraction. The module of an a.p. function is the smallest module which contains all the Fourier exponents of the function.

[3] *Sur les équations différentielles linéaires à coefficients presque-périodiques*, Acta Mathematica, vol. 51 (1928), pp. 31–81.

[4] *Homogeneous systems of differential equations with almost periodic coefficients*, Journal of the London Mathematical Society, vol. 8, pp. 283–288.

[5] *Linear differential equations with almost periodic coefficients*, to appear in the Annals of Mathematics.